Solutions to HW #5: Evens
Section 2.2

2. vertex: \((-1, 1)\)  
\[ g(x) = (x+1)^2 + 1 \]

4. vertex: \((-1, -1)\)  
\[ f(x) = (x+1)^2 - 1 \]

28. \[ f(x) = x^2 - 2x - 15 \]
\[ f(x) = (x^2 - 2x + 1) - 15 - 1 \]
\[ f(x) = (x - 1)^2 - 16 \]
vertex: \((1, -16)\)
x-intercepts:
\[ 0 = (x - 1)^2 - 16 \]
\[ (x - 1)^2 = 16 \]
\[ x - 1 = \pm 4 \]
\[ x = -3 \text{ or } x = 5 \]
y-intercept:
\[ f(0) = 0^2 - 2(0) - 15 = -15 \]
The axis of symmetry is \(x = 1\).

\[ f(x) = x^2 - 2x - 15 \]

Domain: \((-\infty, \infty)\)

Range: \([-16, \infty)\)
40. \( f(x) = 2x^2 - 8x - 3 \)
   
   a. \( a = 2 \). The parabola opens upward and has a minimum value.
   
   b. \( x = \frac{-b}{2a} = \frac{8}{4} = 2 \)
   
   \( f(2) = 2(2)^2 - 8(2) - 3 \)
   
   \( = 8 - 16 - 3 = -11 \)
   
   The minimum is \(-11\) at \( x = 2 \).
   
   c. Domain: \((-\infty, \infty)\) Range: \([-11, \infty)\)
Maximize the area of the playground with 400 feet of fencing.
Let $x$ be the length of the rectangle. Let $y$ be the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express $y$ in terms of $x$.

$$
2x + 3y = 400
$$

$$
3y = 400 - 2x
$$

$$
y = \frac{400 - 2x}{3}
$$

$$
y = \frac{400}{3} - \frac{2}{3}x
$$

We need to maximize $A = xy = x \left( \frac{400}{3} - \frac{2}{3}x \right)$.

Rewrite $A$ as a function of $x$.

$$
A(x) = x \left( \frac{400}{3} - \frac{2}{3}x \right) = -\frac{2}{3}x^2 + \frac{400}{3}x
$$

Since $a = -\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$
x = -\frac{b}{2a} = -\frac{\frac{400}{3}}{2 \left( -\frac{2}{3} \right)} = -\frac{400}{4} = 100.
$$

When the length $x$ is 100, the width $y$ is

$$
y = \frac{400}{3} - \frac{2}{3}x = \frac{400}{3} - \frac{2}{3}(100) = \frac{200}{3} = 66\frac{2}{3}.
$$

The dimensions of the rectangular playground with maximum area are 100 feet by $66\frac{2}{3}$ feet. This gives an area of $100 \cdot 66\frac{2}{3} = 6666\frac{2}{3}$ square feet.
94. We know \( (h,k) = (-3, -4) \), so the equation is of the form 
\[
    f(x) = a(x-h)^2 + k \\
    = a(x-(-3))^2 + (-1) \\
    = a(x+3)^2 - 1 
\]

We use the point \((-2, -3)\) on the graph to determine the value of \(a\): 
\[
    f(x) = a(x+3)^2 - 1 \\
    -3 = a(-2+3)^2 - 1 \\
    -3 = a(1)^2 - 1 \\
    -3 = a - 1 \\
    -2 = a 
\]

Thus, the equation of the parabola is 
\[
    f(x) = -2(x+3)^2 - 1 .
\]
2.3

62. \( f(x) = -3x^3(x-1)^2(x+3) \)

a. Since \( a_n < 0 \) and \( n \) is even, \( f(x) \) falls to the left and the right.

b. \( x = 0, x = 1, x = -3 \)
The roots at 0 and -3 have odd multiplicity so \( f(x) \) crosses the x-axis at those points.
The root at 1 has even multiplicity so \( f(x) \) touches the axis at (1, 0).

c. \( f(0) = -3(0)^3(0-1)^2(0+3) = 0 \)
The y-intercept is 0

d. \( f(-x) = 3x^3(-x-1)^2(-x+3) \)
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and \( 2 \leq 6 - 1 \).

\[ 
\begin{array}{c}
\text{\( y \uparrow \)} \\
\text{500} \\
\text{\( y \downarrow \)} \\
\text{5} \\
\text{\( x \uparrow \)} \\
\end{array}
\]
\[ f(x) = -3x^3(x-1)^2(x+3) \]

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2.4

2. \( \frac{x+5}{x-2} \)

\[
\begin{align*}
x^2 + 3x - 10 \\
x^2 - 2x \\
5x \\
5x - 10 \\
0
\end{align*}
\]
The answer is \( x + 5 \).