HW #8 Solution

7.37

a  \( p = .3; \quad SE (\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.3(.7)}{100}} = .0458 \)

b  \( p = .1; \quad SE (\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.1(.9)}{400}} = .015 \)

c  \( p = .6; \quad SE (\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.6(.4)}{250}} = .031 \)

7.38

For each of the three binomial distributions, calculate \( np \) and \( nq \):

a  \( np = 2.5 \) and \( nq = 47.5 \)

b  \( np = 7.5 \) and \( nq = 67.5 \)

c  \( np = 247.5 \) and \( nq = 2.5 \)

The normal approximation to the binomial distribution is only appropriate for part b, when \( n = 75 \) and \( p = .1 \).

7.39

a  Since \( \hat{p} \) is approximately normal, with standard deviation \( SE (\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.4(6)}{75}} = .0566 \), the probability of interest is

\[
P(\hat{p} \leq .43) = P(z \leq \frac{.43 - .4}{.0566}) = P(z \leq .53) = .7019
\]

b  The probability is approximated as

\[
P(.35 \leq \hat{p} \leq .43) = P \left( \frac{.35 - .4}{.0566} \leq z \leq \frac{.43 - .4}{.0566} \right) = P(-.88 \leq z \leq .53) = .7019 - .1894 = .5125
\]

7.43

a  For \( n = 100 \) and \( p = .19 \), \( np = 19 \) and \( nq = 81 \) are both greater than 5. Therefore, the normal approximation will be appropriate, with mean \( p = .19 \) and \( SE = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.19)(.81)}{100}} = .0392 \).

b  \( P(\hat{p} > .25) = P \left( z > \frac{.25 - .19}{.0392} \right) = P(z > 1.53) = 1 - .9370 = .0630 \)

c  \( P(.25 < \hat{p} < .30) = P \left( \frac{.25 - .19}{.0392} < z < \frac{.30 - .19}{.0392} \right) = P(1.53 < z < 2.80) = .9974 - .9370 = .0604 \)

d  The value \( \hat{p} = .30 \) lies \( z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.30 - .19}{.0392} = 2.80 \) standard deviations from the mean. Also, \( P(\hat{p} \geq .30) = P(z \geq 2.80) = 1 - .9974 = .0026 \). This is an unlikely occurrence, assuming that \( p = .19 \). Perhaps the sampling was not random, or the 19% figure is not correct.

7.47

a  The random variable \( \hat{p} \), the sample proportion of consumers who like nuts or caramel in their chocolate, has a binomial distribution with \( n = 200 \) and \( p = .75 \). Since \( np = 150 \) and \( nq = 50 \) are both greater than 5, this binomial distribution can be approximated by a normal distribution with mean \( p = .75 \) and \( SE = \sqrt{\frac{.75(.25)}{200}} = .0306 \)

b  \( P(\hat{p} > .80) = P \left( z > \frac{.80 - .75}{.0306} \right) = P(z > 1.63) = 1 - .9484 = .0516 \)

c  From the Empirical Rule (and the general properties of the normal distribution), approximately 95% of the measurements will lie within 2 (or 1.96) standard deviations of the mean:

\[
p \pm 2SE \quad \Rightarrow \quad .75 \pm 2(.03062) = .75 \pm .06 \quad \text{or} \quad .69 \text{ to } .81
\]

8.3

For the estimate of \( \mu \) given as \( \bar{X} \), the margin of error is \( 1.96 \ SE = 1.96 \frac{\sigma}{\sqrt{n}} \).

a  \( 1.96 \sqrt{\frac{.2}{30}} = .160 \)

b  \( 1.96 \sqrt{\frac{.9}{30}} = .339 \)

c  \( 1.96 \sqrt{\frac{.5}{30}} = .438 \)
Refer to Exercise 8.3. As the population variance \( \sigma^2 \) increases, the margin of error also increases.

The margin of error is \( 1.96 \frac{SE}{n} = 1.96 \frac{\sigma}{\sqrt{n}} \), where \( \sigma \) can be estimated by the sample standard deviation \( s \) for large values of \( n \).

\[
\begin{align*}
\text{a} & \quad 1.96 \sqrt{\frac{4}{50}} = 0.554 \\
\text{b} & \quad 1.96 \sqrt{\frac{4}{500}} = 0.175 \\
\text{c} & \quad 1.96 \sqrt{\frac{4}{5000}} = 0.055
\end{align*}
\]

Refer to Exercise 8.5. As the sample size \( n \) increases, the margin of error decreases.

The point estimate for \( p \) is given as \( \hat{p} = \frac{x}{n} = 0.51 \) and the margin of error is approximately

\[
1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{0.5(0.5)}{900}} = 0.0327
\]

The sampling error was reported by using the maximum margin of error using \( p = 0.5 \), and by rounding off to the nearest percent:

\[
1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{0.5(0.5)}{900}} = 0.0327 \text{ or } \pm 3\%
\]

A point estimate for the mean length of time is \( \bar{x} = 19.3 \), with margin of error \( 1.96 \frac{SE}{\sqrt{n}} \approx 1.96 \frac{s}{\sqrt{n}} = 1.96 \frac{5.2}{\sqrt{30}} = 1.86 \)

Similar to Exercise 8.22, with a 90% confidence interval for \( \mu \) given as

\[
\bar{x} \pm 1.645 \frac{s}{\sqrt{n}}
\]

where \( s \) can be estimated by the sample standard deviation \( s \) for large values of \( n \).

\[
\begin{align*}
\text{a} & \quad 0.84 \pm 1.645 \sqrt{\frac{0.86}{125}} = 0.84 \pm 0.043 \text{ or } 0.797 < \mu < 0.883 \\
\text{b} & \quad 21.9 \pm 1.645 \sqrt{\frac{3.44}{50}} = 21.9 \pm 0.431 \text{ or } 21.469 < \mu < 22.331 \\
\text{c} & \quad \text{Intervals constructed in this manner will enclose the true value of } \mu \text{ 90% of the time in repeated sampling. Hence, we are fairly confident that these particular intervals will enclose } \mu \text{.}
\end{align*}
\]

The time to complete an online order is probably not mound-shaped. The minimum value of \( x \) is zero, and there is an average time of \( \mu = 4.5 \), with a standard deviation of \( \sigma = 2.7 \). If we calculate \( \mu - 2\sigma = -9 \), leaving no possibility for a measurement to fall more than two standard deviations below the mean. For a mound-shaped distribution, approximately 2.5% should fall in that range. The distribution is probably skewed to the right.

Since \( n \) is large, the Central Limit Theorem ensures that the sample mean \( \bar{x} \) is approximately normal, and the standard normal distribution can be used to construct a confidence interval for \( \mu \).

The 95% confidence interval for \( \mu \) is

\[
\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} = 4.5 \pm 1.96 \frac{2.7}{\sqrt{50}} = 4.5 \pm 0.478 \text{ or } 4.022 < \mu < 4.978
\]

The 99% confidence interval for \( \mu \) is

\[
\bar{x} \pm 2.58 \frac{s}{\sqrt{n}} = 98.25 \pm 2.58 \frac{0.73}{\sqrt{130}} = 98.25 \pm 0.165 \text{ or } 98.085 < \mu < 98.415
\]

Since the possible values for \( \mu \) given in the confidence interval does not include the value \( \mu = 98.6 \), it is not likely that the true average body temperature for healthy humans is 98.6, the usual average temperature cited by physicians and others.