9.3   **a**   The critical value that separates the rejection and nonrejection regions for a right-tailed test based on a z-statistic will be a value of z (called z_α) such that $P(z > z_\alpha) = \alpha = .01$. That is, $z_{.01} = 2.33$ (see the figure below). The null hypothesis H_0 will be rejected if $z > 2.33$.
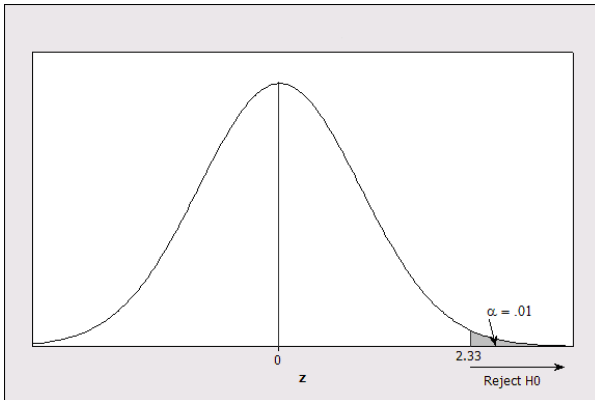
b   For a two-tailed test with $\alpha = .05$, the critical value for the rejection region cuts off $\alpha/2 = .025$ in the two tails of the z distribution in Figure 9.2, so that $z_{.025} = 1.96$. The null hypothesis H_0 will be rejected if $z > 1.96$ or $z < -1.96$ (which you can also write as $|z| > 1.96$).
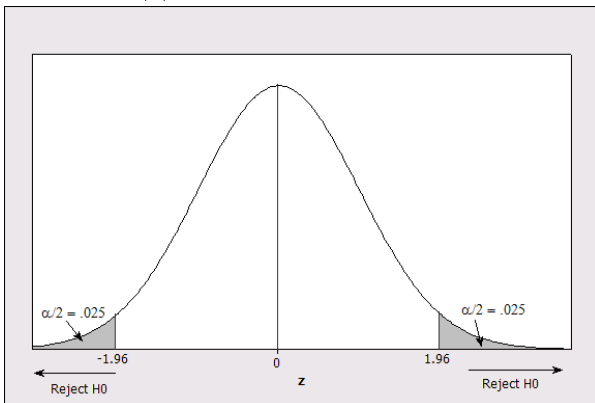

c   Similar to part **a**, with the rejection region in the lower tail of the z distribution. The null hypothesis H_0 will be rejected if $z < -2.33$.

d   Similar to part **b**, with $\alpha/2 = .005$. The null hypothesis H_0 will be rejected if $z > 2.58$ or $z < -2.58$ (which you can also write as $|z| > 2.58$).

9.4   **a**   The p-value for a right-tailed test is the area to the right of the observed test statistic $z = 1.15$ or
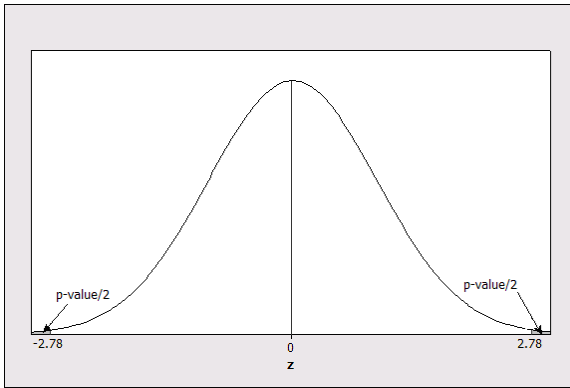
$$p\text{-value} = P(z > 1.15) = 1 - .8749 = .1251$$

This is the shaded area in the figure below.
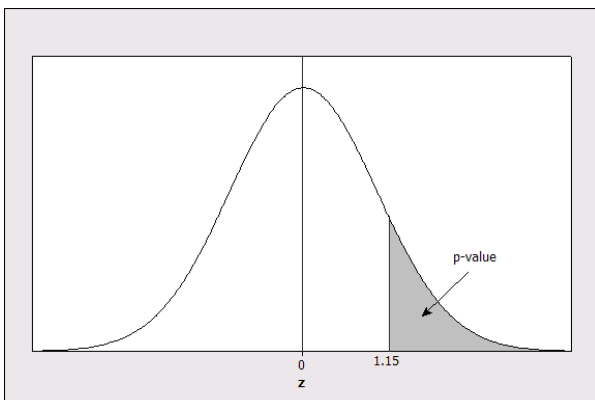
b For a two-tailed test, the *p*-value is the probability of being as large or larger than the observed test statistic *in either tail of the sampling distribution*. As shown in the figure below, the *p*-value for $z = -2.78$ is

$$p\text{-value} = P(|z| > 2.78) = 2(.0027) = .0054$$

c The *p*-value for a left-tailed test is the area to the left of the observed test statistic $z = -1.81$ or

$$p\text{-value} = P(z < -1.81) = .0351$$

9.6 In this exercise, the parameter of interest is μ, the population mean. The objective of the experiment is to show that the mean exceeds 2.3.

a We want to prove the alternative hypothesis that μ is, in fact, greater then 2.3. Hence, the alternative hypothesis is

$$H_a : \mu > 2.3$$

and the null hypothesis is

$$H_0 : \mu = 2.3 .$$

b The best estimator for μ is the sample average $\bar{x}$, and the test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

which represents the distance (measured in units of standard deviations) from $\bar{x}$ to the hypothesized mean μ. Hence, if this value is large in absolute value, one of two conclusions may be drawn. Either a very unlikely event has occurred, or the hypothesized mean is incorrect. Refer to part **a**. If $\alpha = .05$, the critical value of z that separates the rejection and non-rejection regions will be a value (denoted by z_0) such that

$$P(z > z_0) = \alpha = .05$$

That is, $z_0 = 1.645$ (see below). Hence, H_0 will be rejected if $z > 1.645$.
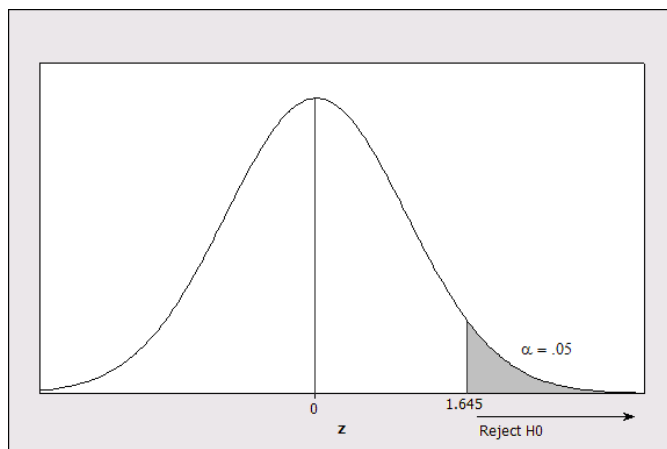
c The standard error of the mean is found using the sample standard deviation s to approximate the population standard deviation σ:

$$SE = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}} = \frac{.29}{\sqrt{35}} = .049$$

d   To conduct the test, calculate the value of the test statistic using the information contained in the sample. Note that the value of the true standard deviation, σ , is approximated using the sample standard deviation s.

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \approx \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{2.4 - 2.3}{.049} = 2.04$$

The observed value of the test statistic, $z = 2.04$, falls in the rejection region and the null hypothesis is rejected. There is sufficient evidence to indicate that $\mu > 2.3$.

9.13   **a-b** We want to test the null hypothesis that μ is, in fact, 80% against the alternative that it is not:

$$H_0 : \mu = 80 \quad \text{versus} \quad H_a : \mu \neq 80$$

Since the exercise does not specify $\mu < 80$ or $\mu > 80$, we are interested in a two directional alternative, $\mu \neq 80$.

c   The test statistic is

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \approx \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{79.7 - 80}{.8/\sqrt{100}} = -3.75$$

The rejection region with $\alpha = .05$ is determined by a critical value of z such that

$$P(z < -z_0) + P(z > z_0) = \frac{\alpha}{2} + \frac{\alpha}{2} = .05$$

This value is $z_0 = 1.96$ (see the figure in Exercise 9.3**b**). Hence, H$_0$ will be rejected if $z > 1.96$ or $z < -1.96$. The observed value, $z = -3.75$, falls in the rejection region and H$_0$ is rejected. There is sufficient evidence to refute the manufacturer's claim. The probability that we have made an incorrect decision is $\alpha = .05$.

9.17   The hypothesis to be tested is

$$H_0 : \mu = 5.97 \quad \text{versus} \quad H_a : \mu > 5.97$$

and the test statistic is

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \approx \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{9.8 - 5.97}{1.95/\sqrt{31}} = 10.94$$

with p-value $= P(z > 10.94) < 1 - .9998 = .0002$ (or p-value ≈ 0). Since the p-value is less than .05, the null hypothesis is rejected . There is sufficient evidence to indicate that the average diameter of the tendon for patients with AT is greater than 5.97 mm.

Alternately, you can use the critical value approach: We will reject the null if z>1.645. Since z=10.94 > 1.645, we decide to reject the null. At the 0.05 level of significance, there is sufficient evidence that the true mean diameter of the tendon for patients with AT is greater than 5.97 mm.