HW #3: Solution to Evens
Statistics 1, Sections 4 and 6, Fall 2008

4.2  a  It is given that \( P(E_1) = P(E_2) = 1.5 \) and \( P(E_3) = .40 \). Since \( \sum_s P(E_s) = 1 \), we know that

\[
P(E_4) + P(E_5) = 1 - 1.5 - 1.5 - .40 = .30 \tag{i}
\]

Also, it is given that

\[
P(E_4) = 2P(E_5) \tag{ii}
\]

We have two equations in two unknowns which can be solved simultaneously for \( P(E_4) \) and \( P(E_5) \). Substituting equation (ii) into equation (i), we have

\[
2P(E_5) + P(E_5) = .3
\]

\[
3P(E_5) = .3 \text{ so that } P(E_5) = .1
\]

Then from (i), \( P(E_4) + .1 = .3 \) and \( P(E_4) = .2 \).

b  To find the necessary probabilities, sum the probabilities of the simple events:

\[
P(A) = P(E_1) + P(E_1) + P(E_4) = .15 + .4 + .2 = .75
\]

\[
P(B) = P(E_2) + P(E_5) = .15 + .4 = .55
\]

c-d  The following events are in either A or B or both: \( \{E_2, E_3, E_4, E_5\} \). Only event \( E_3 \) is in both A and B.

4.42  Each simple event is equally likely, with probability \( 1/5 \).

a  \( A^C = \{E_2, E_4, E_5\} \)  \( P(A^C) = 3/5 \)

b  \( A \cap B = \{E_1\} \)  \( P(A \cap B) = 1/5 \)

c  \( B \cap C = \{E_4\} \)  \( P(B \cap C) = 1/5 \)

d  \( A \cup B = S = \{E_1, E_2, E_3, E_4, E_5\} \)  \( P(A \cup B) = 1 \)

e  \( B | C = \{E_4\} \)  \( P(B | C) = 1/2 \)

f  \( A | B = \{E_1\} \)  \( P(A | B) = 1/4 \)

g  \( A \cup B \cup C = S \)  \( P(A \cup B \cup C) = 1 \)

h  \( (A \cap B)^C = \{E_2, E_3, E_4, E_5\} \)  \( P(A \cap B)^C = 4/5 \)
The two-way table in the text gives probabilities for events $A$, $A^c$, $B$, $B^c$ in the column and row marked “Totals”. The interior of the table contains the four two-way intersections as shown below.

<table>
<thead>
<tr>
<th>$A \cap B$</th>
<th>$A \cap B^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^c \cap B$</td>
<td>$A^c \cap B^c$</td>
</tr>
</tbody>
</table>

The necessary probabilities can be found using various rules of probability if not directly from the table.

a. $P(A) = .4$

b. $P(B) = .37$

c. $P(A \cap B) = .10$

d. $P(A \cup B) = .4 + .37 - .10 = .67$

e. $P(A^c) = 1 - .4 = .6$

f. $P(A \cup B)^c = 1 - P(A \cup B) = 1 - .67 = .33$

g. $P(A^c \cap B)^c = 1 - P(A^c \cap B) = .90$

h. $P(A \mid B) = P(A \cap B) / P(B) = .1 / .37 = .27$

i. $P(B \mid A) = P(A \cap B) / P(A) = .1 / .4 = .25$

Similar to Exercise 4.56.

a. $P(F) = .35 + .36 = .71$

b. $P(G) = .20 + .09 = .29$

c. $P(F \mid M) = P(F \cap M) / P(M) = .35 / .55 = .63$

d. $P(F \mid W) = P(F \cap W) / P(W) = .36 / .45 = .80$

e. $P(M \mid F) = P(M \cap F) / P(F) = .35 / .71 = .49$

e. $P(W \mid G) = P(W \cap G) / P(G) = .09 / .29 = .31$