Practice Problems for Exam 2  
Note: This is not a comprehensive list of the types of problems for the exam. Be sure to review all of sections 4.6-6.3 that we covered in class and homeworks #4-7. You are only responsible for the parts of 6.3 that deal with the standard normal random variable (which has mean 0 and variance 1).

1) The number of emergency calls, $X$, that Dr. Suess receives during a typical day has the following probability distribution:

<table>
<thead>
<tr>
<th>Number of calls, $x$</th>
<th>Probability $p(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>1</td>
<td>0.28</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
</tr>
</tbody>
</table>

   a) What is the probability Dr. Suess receives no emergency calls in a day?
   b) Calculate the mean and standard deviation of $X$.
   c) Explain the meaning of the mean calculated in part (c) without using statistical jargon.

2) A machine manufactures a large number of bolts of a certain type. It is known that 15% of all bolts are defective. If 10 bolts are randomly selected, find the probability

   a) Exactly 3 are defective
   b) At least 2 are defective
   c) What are the mean and standard deviation of the number of defective bolts among a sample of 10?
   d) If 200 bolts are randomly selected, would it be unusual to observe 50 defective bolts?

3) A busy person is going to buy two different snacks from a vending machine by randomly selecting them. There are four choices: bananas (B), apples (A), Snickers candy bars (S), and cookies (C).

   a) List the sample space, $S$, for this experiment using the given abbreviations.
   b) Let the random variable $X$ represent the number of fruits the person gets in the two randomly selected snacks. Give the probability distribution of $X$.
   c) Given that one of the person’s randomly selected snacks is a fruit, what is the probability that both are fruits.

4) Which of the following random variables are binomial random variables?

   a) A small class consists of 10 students. 7 are men and 3 are women. Randomly select 4 students. Let $X$ = the number of women selected.
   b) $X$ = the number of red cars entering a parking garage in 1 minute
   c) You play 20 games of roulette, betting on red each time. $X$ = the number of games you win
   d) Randomly select 100 Americans. $X$ = the number who have diabetes.
   e) Toss a pair of fair dice five times. $X$ = the number of times doubles appears.
   f) Toss a pair of fair dice until doubles appears. $X$ = the number of tosses
5)  
   a) The probability your burglar alarm does not detect an intruder is 1%. The probability your dog does not detect the intruder is 5%. What is the probability that neither the burglar alarm nor the dog detects the intruder if we can assume these events to be independent?

   b) Suppose 0.1% of the population has Disease X. If a person has Disease X, the test will correctly indicate this 97% of the time. For people who do not have the disease, the test will incorrectly indicate that they have Disease X 5% of the time. What is the probability a randomly selected person

      i) Has Disease X and tests positive for it?
      ii) Does not have disease X and tests positive for it?

6) You play a game where a 3-digit whole number from 0 to 999 is randomly selected. Each number is regarded as having 3 digits, so, for example 1 = 001 and 23=023. You pay $1 to play (the casino or “house” keeps your $1, regardless of the outcome). You win $50 if all three digits in the number are the same. You win $10 if exactly 2 digits are the same (for example, 223 or 191). If all the digits are different you win nothing.

   a) What is the probability distribution of the amount won or lost on this game?
   b) What is the expected value of the amount won or lost?
   c) Interpret the expected value from part (b) without using any statistical jargon.
   d) How would you use the Law of Large Numbers to estimate the probability of winning $50 in this game?

7) Let Z denote the standard normal random variable, which has mean 0 and standard deviation 1. Compute the following probabilities:

   a) P(Z > -0.85)
   b) P(Z ≥ -0.85)
   c) P(0.40 < Z <1.30)
   d) P(-0.30 < Z < 0.90)
   e) P(-1.53 < Z < -0.45)

   Find the constant c for which
   f) P(Z ≤ c) = 0.7967
   g) P(Z > c) = 0.12
   h) P(Z ≥ c) = 0.65
   i) P(-c < Z < c) = 0.90
   j) Find the 90th percentile of the Z-distribution.