1. 1st period income is either saved or consumed. Whatever is saved will be consumed at the second period.
If I use $a_t$ to denote saving,
\[ x_t = c_{1,t} + a_t \]
\[ a_t(1 + r) = c_{2,t+1} \]
combining these two:
\[ x_t = c_{1,t} + \frac{c_{2,t+1}}{1 + r} \]

graphing this:

Household’s utility max. problem:
max. \( \log(C_{1,t}) + \frac{1}{1 + \rho} \log(C_{2,t+1}) \) subject to \( x_t = c_{1,t} + \frac{c_{2,t+1}}{1 + r} \)
Using Lagrangian,
\[ \mathcal{L} = \log(C_{1,t}) + \frac{1}{1 + \rho} \log(C_{2,t+1}) + \lambda \left[ x_t - c_{1,t} - \frac{c_{2,t+1}}{1 + r} \right] \]
FOC:
\[ \frac{1}{C_{1,t}} - \lambda = 0 \]
\[ \frac{1}{1 + \rho C_{2,t+1}} \frac{1}{1 + r} - \lambda = 0 \]
Combining these two FOC’s gives the Euler Equation;
\[ c_{1,t} = \left( \frac{1 + \rho}{1 + r} \right) c_{2,t+1} \]
According to the Euler Equation, if \( \rho = r \), we equalize the consumption in both periods. This means my discount parameter should be the same as the interest I earn from saving.

To observe 20% saving among households, I need \( \rho = 3 \).
Remember that \( x_t = c_{1,t} + a_t \). 20% saving means \( a_t \) should be equal to 0.2\( x_t \)
So, \( x_t = c_{1,t} + 0.2x_t \) and that yields \( c_{1,t} = 0.8x_t \)

Now, combining the budget constraint and the Euler equation, write
\[
x_t = c_{1,t} + \frac{c_{1,t}}{1 + \rho}
\]
which yields \( \rho = 3 \) when \( c_{1,t} = 0.8x_t \) is plugged into it and simplified.

2. Maximize \( \ln(C_{1,i}) + \beta \ln(C_{2,i}) \) subject to \( y_{1,i} + \frac{y_{2,i}}{R} = C_{1,i} + \frac{C_{2,i}}{R} \)
   a. The Euler equation from this maximization becomes
   \[
   \frac{C_{2,i}}{C_{1,i}} = \beta R
   \]
   Plugging it back into the budget constraint will give us the solution for first period consumption:
   \[
   C_{1,i} = \frac{y_{1,i} + \frac{y_{2,i}}{R}}{1 + \beta}
   \]
   b. \( C^T = \sum_{i=1}^{N} C_{1,i} = \sum_{i=1}^{N} \frac{y_{1,i} + \frac{y_{2,i}}{R}}{1 + \beta} \) using superscript \( T \) to denote sum or total.
   c. \( y_{1,i} = C_{1,i} + S_{1,i} \) hence,
   \[
   S_{1,i} = y_{1,i} - C_{1,i}
   \]
   \[
   S_{1,i} = y_{1,i} - \frac{y_{1,i} + \frac{y_{2,i}}{R}}{1 + \beta}
   \]
   \[
   S_{1,i} = \frac{\beta y_{1,i} + \frac{y_{2,i}}{R}}{1 + \beta}
   \]

fraction \( \lambda \) has
   \[
   S_1 = \frac{\beta y_1}{1 + \beta}
   \]
and fraction \( 1-\lambda \) has
   \[
   S_2 = \frac{y_2}{R(1 + \beta)}
   \]
setting borrowing and lending equal to each other,
\[
\lambda \frac{\beta y_1}{1 + \beta} = (1 - \lambda) \frac{y_2}{R(1 + \beta)}
\]

Solving for \(R^*\), we get

\[
R^* = \frac{(1 - \lambda) \ y_2}{\lambda \ \beta y_1}
\]

Hence, when I discount future more, \(R^*\) decreases. When a higher proportion of people have just first period income, \(R^*\) decreases and when a higher proportion of people have just second period income, \(R^*\) increases.

Also, as second period income is higher, \(R^*\) goes up and it goes down as first period income is higher.