# DeMorgan’s Theorems

Handout

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# DeMorgan's Theorems

DeMorgan's Theorems are two additional simplification techniques that can be used to simplify Boolean expressions.  Again, the simpler the Boolean expression, the simpler the resulting logic.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

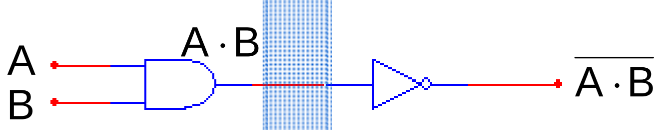
$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

# DeMorgan’s Theorem #1

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

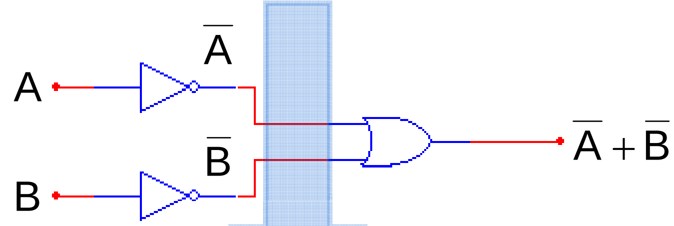
*Proof*

$\overline{A \cdot B}$




| A | B | $A \cdot B$ | $\overline{A \cdot B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$\overline{A} + \overline{B}$




| A | B | $\overline{A}$ | $\overline{B}$ | $\overline{A} + \overline{B}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |

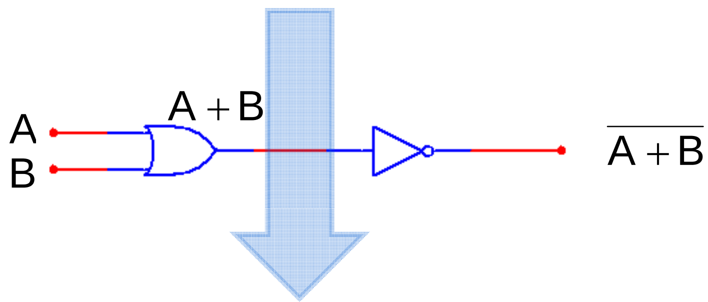
*The truth-tables are equal; therefore, the Boolean equations must be equal.*

# DeMorgan's Theorem #2

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

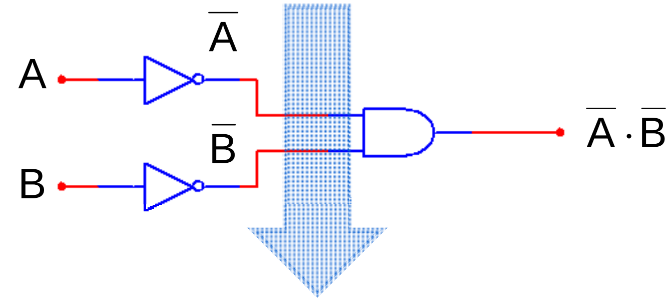
*Proof*

$\overline{A + B}$




| A | B | $A + B$ | $\overline{A + B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

$\overline{A} \cdot \overline{B}$




| A | B | $\overline{A}$ | $\overline{B}$ | $\overline{A} \cdot \overline{B}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |

*The truth-tables are equal; therefore, the Boolean equations must be equal.*

# Summary

## Boolean & DeMorgan's Theorems

1) $X \cdot 0 = 0$
2) $X \cdot 1 = X$
3) $X \cdot X = X$
4) $X \cdot \overline{X} = 0$
5) $X + 0 = X$
6) $X + 1 = 1$
7) $X + X = X$
8) $X + \overline{X} = 1$
9) $\overline{\overline{X}} = X$

10A) $X \cdot Y = Y \cdot X$
10B) $X + Y = Y + X$

Commutative Law

11A) $X(YZ) = (XY)Z$
11B) $X + (Y + Z) = (X + Y) + Z$

Associative Law

12A) $X(Y + Z) = XY + XZ$
12B) $(X + Y)(W + Z) = XW + XZ + YW + YZ$

Distributive Law

13A) $X + \overline{X}Y = X + Y$
13B) $\overline{X} + XY = \overline{X} + Y$
13C) $X + \overline{X}\,\overline{Y} = X + \overline{Y}$
13D) $\overline{X} + X\overline{Y} = \overline{X} + \overline{Y}$

Consensus Theorem

14A) $\overline{X\,Y} = \overline{X} + \overline{Y}$
14B) $\overline{X + Y} = \overline{X}\;\overline{Y}$

DeMorgan's

# DeMorgan Shortcut

## BREAK THE LINE, CHANGE THE SIGN

Break the ***LINE*** over the two variables,
and change the ***SIGN*** directly under the line.

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

For Theorem #14A, break the line, and change the AND function to an OR function. Be sure to keep the lines over the variables.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

For Theorem #14B, break the line, and change the OR function to an AND function. Be sure to keep the lines over the variables.

# DeMorgan’s: Example #1

*Example*

Simplify the following Boolean expression and note the Boolean or DeMorgan’s theorem used at each step. Put the answer in SOP form.

$$F_1 = \overline{(\overline{X \cdot \overline{Y}}) \cdot (\overline{Y} + Z)}$$

# DeMorgan’s: Example #1

*Example*

Simplify the following Boolean expression and note the Boolean or DeMorgan’s theorem used at each step. Put the answer in SOP form.

$$F_1 = \overline{\overline{(X \cdot \overline{\overline{Y}})} \cdot (\overline{Y} + Z)}$$

*Solution*

$$F_1 = \overline{\overline{(X \cdot \overline{\overline{Y}})} \cdot (\overline{Y} + Z)}$$

$$F_1 = \overline{\overline{(X \cdot \overline{\overline{Y}})}} + \overline{(\overline{Y} + Z)}$$ ; Theorem #14A

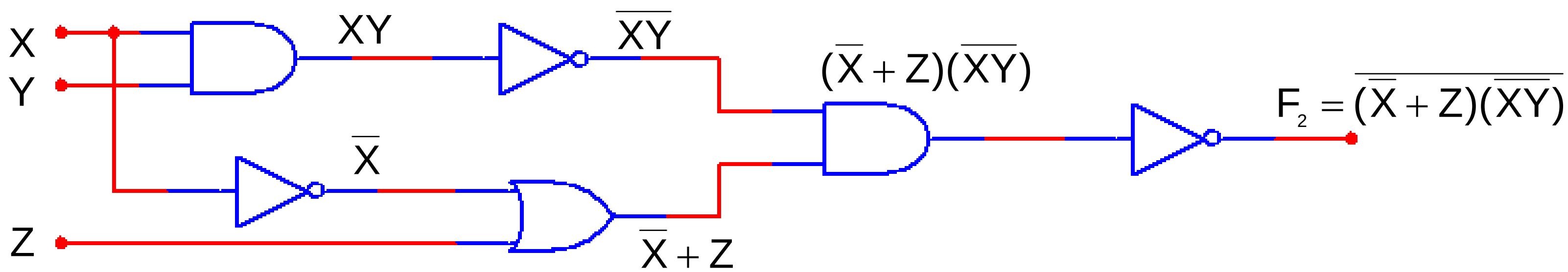
$$F_1 = (X \cdot \overline{Y}) + (\overline{\overline{Y}} \cdot \overline{Z})$$ ; Theorem #9 & #14B

$$F_1 = (X \cdot \overline{Y}) + (Y \cdot \overline{Z})$$ ; Theorem #9

$$F_1 = X\overline{Y} + Y\overline{Z}$$ ; Rewritten without AND symbols and parentheses

# DeMorgan's: Example #2

So, where would such an odd Boolean expression come from? Take a look at the VERY poorly designed logic circuit shown below. If you were to analyze this circuit to determine the output function $F_2$, you would obtain the results shown.




*Example*

Simplify the output function $F_2$. Be sure to note the Boolean or DeMorgan's theorem used at each step. Put the answer in SOP form.

# DeMorgan's: Example #2

*Solution*

$$F_2 = \overline{(\overline{X} + Z)(\overline{XY})}$$

$$F_2 = \overline{(\overline{X} + Z)} + (\overline{\overline{XY}})$$ ; Theorem #14A

$$F_2 = \overline{(\overline{X} + Z)} + (XY)$$ ; Theorem #9

$$F_2 = (\overline{\overline{X}}\,\overline{Z}) + (XY)$$ ; Theorem #14B

$$F_2 = (X\,\overline{Z}) + (XY)$$ ; Theorem #9

$$F_2 = X\,\overline{Z} + X\,Y$$ ; Rewritten without AND symbols