DeMorgan’s Theorems

Handout

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DeMorgan’s Theorems

DeMorgan’s Theorems are two additional simplification techniques that can be used to simplify Boolean expressions. Again, the simpler the Boolean expression, the simpler the resulting logic.

\[ A + B = \overline{A} \cdot \overline{B} \]

\[ A \cdot B = \overline{A} + \overline{B} \]
DeMorgan’s Theorem #1

\[ A \cdot B = \overline{A} + \overline{B} \]

**Proof**

The truth-tables are equal; therefore, the Boolean equations must be equal.
DeMorgan’s Theorem #2

\[ A + B = \overline{A} \cdot \overline{B} \]

**Proof**

The truth-tables are equal; therefore, the Boolean equations must be equal.
Summary

Boolean & DeMorgan’s Theorems

1) \( X \cdot 0 = 0 \)
2) \( X \cdot 1 = X \)
3) \( X \cdot X = X \)
4) \( X \cdot \overline{X} = 0 \)
5) \( X + 0 = X \)
6) \( X + 1 = 1 \)
7) \( X + X = X \)
8) \( X + \overline{X} = 1 \)
9) \( \overline{X} = X \)

10A) \( X \cdot Y = Y \cdot X \)  
10B) \( X + Y = Y + X \) \text{ Commutative Law}

11A) \( X(YZ) = (XY)Z \)  
11B) \( X + (Y + Z) = (X + Y) + Z \) \text{ Associative Law}

12A) \( X(Y + Z) = XY + XZ \)  
12B) \( (X + Y)(W + Z) = XW + XZ + YW + YZ \) \text{ Distributive Law}

13A) \( X + \overline{XY} = X + Y \)  
13B) \( \overline{X} + XY = \overline{X} + Y \) \text{ Consensus Theorem}
13C) \( X + \overline{XY} = X + \overline{Y} \)
13D) \( \overline{X} + X\overline{Y} = \overline{X} + \overline{Y} \)

14A) \( \overline{XY} = \overline{X} + \overline{Y} \)  
14B) \( \overline{X + Y} = \overline{X} \overline{Y} \) \text{ DeMorgan’s Theorem}
DeMorgan Shortcut

BREAK THE LINE, CHANGE THE SIGN

Break the \textit{LINE} over the two variables, and change the \textit{SIGN} directly under the line.

\[ \overline{A \cdot B} = \overline{A} + \overline{B} \]

For Theorem #14A, break the line, and change the AND function to an OR function. Be sure to keep the lines over the variables.

\[ \overline{A + B} = \overline{A} \cdot \overline{B} \]

For Theorem #14B, break the line, and change the OR function to an AND function. Be sure to keep the lines over the variables.
DeMorgan’s: Example #1

Example

Simplify the following Boolean expression and note the Boolean or DeMorgan’s theorem used at each step. Put the answer in SOP form.

\[ F_1 = (X \cdot \overline{Y}) \cdot (\overline{Y} + Z) \]
DeMorgan’s: Example #1

Example

Simplify the following Boolean expression and note the Boolean or DeMorgan’s theorem used at each step. Put the answer in SOP form.

\[ F_1 = (X \cdot \overline{Y}) \cdot (\overline{Y} + Z) \]

Solution

\[ F_1 = (X \cdot \overline{Y}) \cdot (\overline{Y} + Z) \]

\[ F_1 = (X \cdot \overline{Y}) + (\overline{Y} + Z) \quad ; \text{Theorem #14A} \]

\[ F_1 = (X \cdot \overline{Y}) + (\overline{Y} \cdot \overline{Z}) \quad ; \text{Theorem #9 & #14B} \]

\[ F_1 = (X \cdot \overline{Y}) + (Y \cdot \overline{Z}) \quad ; \text{Theorem #9} \]

\[ F_1 = X\overline{Y} + Y\overline{Z} \quad ; \text{Rewritten without AND symbols and parentheses} \]
DeMorgan’s: Example #2

So, where would such an odd Boolean expression come from? Take a look at the VERY poorly designed logic circuit shown below. If you were to analyze this circuit to determine the output function $F_2$, you would obtain the results shown.

Example

Simplify the output function $F_2$. Be sure to note the Boolean or DeMorgan’s theorem used at each step. Put the answer in SOP form.

\[ F_2 = (\overline{X + Z})(\overline{XY}) \]
DeMorgan’s: Example #2

Solution

\[ F_2 = (\overline{X + Z})(XY) \]

\[ F_2 = (\overline{X + Z}) + (\overline{XY}) \quad ; \text{Theorem #14A} \]

\[ F_2 = (\overline{X + Z}) + (XY) \quad ; \text{Theorem #9} \]

\[ F_2 = (\overline{X \overline{Z}}) + (XY) \quad ; \text{Theorem #14B} \]

\[ F_2 = (X \overline{Z}) + (XY) \quad ; \text{Theorem #9} \]

\[ F_2 = X \overline{Z} + X Y \quad ; \text{Rewritten without AND symbols} \]