Verschränkung versus Stosszahlansatz: The second law rests on low correlation levels in our cosmic neighborhood

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Outline

Abstract

The extremes of correlation level in the environment are shown to lead to opposite directions of the thermodynamic arrow. In a low-entropy environment where the mean correlation level is low, the **Stosszahlansatz scenario**, the Clausius inequality and the second law are established using general arguments of quantum information theory. For a high-correlation environment, the **Verschränkung** scenario, a model of entangled systems is constructed in which heat flows from the cold to the hot body in direct violation of the second law. This model clearly shows that the second law of thermodynamics is a consequence of quantum statistical dynamics under conditions of low ambient correlations and breaks down under opposite conditions. When the Stosszahlansatz scenario prevails, the second law is valid for systems of any size, although its utility is limited for microscopic systems where fluctuations are not negligible relative to mean values. While the von Neumann entropy and other information-theoretic objects are used extensively in establishing the above results, all conclusions are based on energy flow between systems without any assumption on the thermodynamic interpretation of such objects. The results of this work strongly support the expectation, first expressed by Boltzmann and subsequently elaborated by others, that the second law is an emergent consequence of fundamental dynamics operating in a low-entropy cosmological environment functioning as an ideal information sink.

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- The very large number of physicists who have written on these subjects with little consensus achieved. Some contend that there are no such problems, others believe that the solution to both problems are already known. However, the developments of the past 2-3 decades have convinced most physicists that these issues are both real and objective.
- I believe that the thermodynamic arrow and the second law and the phenomenon of reduction in quantum measurements are consequences of quantum dynamics as already known, and that methods of quantum information theory are best suited to their analysis and resolution.

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- An accelerating universe is an ideal entropy (waste information) dump, allowing the Earth to maintain a low-correlation environment and thermodynamics as we know it.
- We use methods of quantum information and base our conclusions on energy flow only, thereby avoiding controversial issues of interpretation, e.g., of entropic quantities.

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Let the systems be labeled $i = 1, 2, \ldots N$, and the (von Neumann) entropies S^i. The two-body correlation information for the (i, j) pair is defined as $I^{ij} = S^i + S^j - S^{ij}$.

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Strong subadditivity implies $S^i + S^j \leq S^{ik} + S^{jk}$. There are $N(N-1)/2$ such inequalities, which, when aggregated, give

$$[N(N-1)/2]^{-1} \sum_{i<j=1}^{N(N-1)/2} I^{ij} \leq N^{-1} \sum_{i=1}^{N} S^i. \tag{1}$$

Thus $I_{av} \leq S_{av}$, indicating that a small average entropy guarantees a low level of two-body correlations.

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The significance of this result is the **universal** validity of Boltzmann's ***Stosszahlansatz***, for all things large or small, as a likely condition in a low-entropy universe.

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In general, **the Hamiltonian will change during the evolution (causing exchange of work), the system will evolve in a non-unitary manner, and ρ_f will be out of equilibrium.**

We can use the non-negativity of the relative entropy $S(\rho_f \| \rho_i)$ to assert that

$$S(\rho_f \| \rho_i) = \beta_i \Delta U - \Delta S - \beta_i \mathrm{tr}(\rho_f \Delta H) \geq 0, \tag{2}$$

where $\Delta U = U_f - U_i = \mathrm{tr}(\rho_f H_f) - \mathrm{tr}(\rho_i H_i)$, and $\Delta S = S_f - S_i$. This is the *fundamental inequality*.

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But why? Does it, or the fundamental inequality, violate time-reversal invariance?

Let us repeat the above derivation, but this time for a process that starts from an arbitrary state and ends up in equilibrium. It is not difficult to see that instead of Eq. (2) one gets

$$S(\rho_i \| \rho_f) = -\beta_f \Delta U + \Delta S + \beta_f \mathrm{tr}(\rho_i \Delta H) \geq 0, \tag{6}$$

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Eqs. (5) and (7) are nevertheless consistent with equilibrium thermodynamics since, **for any small process whose endpoints are equilibrium states, both (7) and (5) apply, implying that** $dS = \delta Q / T$.

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For a *Stosszahlansatz* universe, Scenario S, Eq. (1) guarantees low correlation levels, so a typical pair of systems A and B, in equilibrium at temperatures T^A and T^B, will be initially uncorrelated. Therefore $\rho_i^{AB} = \rho_i^A \otimes \rho_i^B$ and $S_i^{AB} = S_i^A + S_i^B$.

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Upon interaction, the final state ρ_f^{AB} will in general be correlated so that $S_f^{AB} \leq S_f^A + S_f^B$. Since the pair is isolated, ρ_i^{AB} and ρ_f^{AB} are related unitarily and $S_f^{AB} = S_i^{AB}$.

We therefore find

$$\Delta S^A + \Delta S^B \geq 0 \text{ (Scenario S: systems initially in equilibrium)}. \qquad (8)$$

This equation is a consequence of universal *Stosszahlansatz*. It is the origin of irreversibility in the second law. Recall that the two systems were initially in equilibrium and only exchanged heat. Therefore, we find from Eq. (2) that $\Delta S^A \leq Q^A/T^A$ and $\Delta S^B \leq Q^B/T^B$.

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Combining the latter with $\Delta S^A + \Delta S^B \geq 0$, we find $Q^A/T^A + Q^B/T^B \geq 0$, and using $Q^A + Q^B = 0$ (isolated pair exchanging heat only), we conclude that $Q^A(1/T^A - 1/T^B) \geq 0$.

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Thus Q^A has the same sign as $T^B - T^A$, i.e., **heat flows from the *initially* hotter system to the *initially* colder one**, whence the normal thermodynamic arrow.

Note: Final states are in general not in equilibrium.

We will now establish the **second law** in full generality by proving **Clausius' inequality** under Scenario S, the state of our universe. Consider a system S that undergoes a cyclic process in thermal contact with a series of heat reservoirs, R_j at temperature T_j^R, absorbing Q_j^S from the jth reservoir, and exchanging work as a result of changes in its Hamiltonian.

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The jth process starts with the state $\rho_j^S \rho_j^R$ (Scenario S), where ρ_j^R is a Gibbs state at temperature T_j^R and ρ_j^S is arbitrary. The end state is ρ_j^{SR}.

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Just as in case S above, we find $\Delta S_j^S + \Delta S_j^R \geq 0$, V_j^S notwithstanding, and applying inequality (2) to the *reservoir*, we conclude that $\beta_j^R Q_j^R \geq \Delta S_j^R$, where $\beta_j^R = 1/kT_j^R$ refers to the initial temperature of the reservoir.

Energy exchange between the system and the reservoir is subject to $Q_j^R = -Q_j^S$ since work exchange does not involve the reservoir. Combining the last two inequalities, we find $\beta_j^R Q_j^S + \Delta S_j^S \leq 0$.

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$$\sum_j \beta_j^R Q_j^S \leq 0. \tag{9}$$

This is the Clausius inequality, which is equivalent to the second law. It is a theorem in quantum mechanics for a low-entropy universe.

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Note that the inverse temperature β_j^R refers to the reservoirs, not the system, as in general the system is not at equilibrium as it goes through the cycle. Note also that, as promised, our end results are statement about energy flow and do not involve entropic quantities (which have caused endless arguments in the literature).

Scenario V: breakdown of the second law

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Let the energy spectra of the two systems, $\{E_i^A\}$ and $\{E_i^B\}$, be identical except for a scale factor, i.e., $\mu^A E_i^A = \mu^B E_i^B = \epsilon_i$. The desired state can then be represented as $\rho^{AB} = |\Omega^{AB}\rangle\langle\Omega^{AB}|$, with

$$|\Omega^{AB}\rangle = Z^{-1/2} \sum_i \exp(-\gamma\epsilon_i/2)|i;A\rangle|i;B\rangle, \tag{10}$$

where $|i;A\rangle$ ($|i;B\rangle$) is the ith energy eigenvector of system A (B), γ is a positive number, and $Z^{-1/2}$ is a normalization constant.

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How do we understand this bizarre behavior?

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Scenarios S and V are paradigms, informing us that the second law is not an independent, or inviolable, law of nature. Rather, it is a consequence of quantum mechanics conditioned on a low-correlation environment, and breaks down under the opposite conditions.

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- The reversibility issues that so haunted Boltzmann at the end of the nineteenth century are seen to arise from the initial conditions prevailing in the environment. To his credit, Boltzmann had already guessed that!
- Understanding thermodynamic behavior as a consequence of quantum dynamics conditioned on the environment is a prerequisite to the resolution of the reduction phenomenon in quantum measurements.

This talk is for the most part based on

- M. Hossein Partovi, "Entanglement versus *Stosszahlansatz*: Disappearance of the thermodynamic arrow in a high-correlation environment," *Phys. Rev. E* **77**, 021110 (2008).
- M. Hossein Partovi, "Quantum Thermodynamics," *Phys. Letters* A **137**, 440 (1989).
- M. Hossein Partovi, "Irreversibility, reduction, and entropy increase in quantum measurements," *Phys. Letters* A **137**, 445 (1989).

Further results extending those of the first paper above can be found in

- David Jennings and Terry Rudolph, "Entanglement and the thermodynamic arrow of time," *Phys. Rev. E* **81**, 061130 (2010).

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