Verschränkung vs. *Stosszahlansatz*: Correlation Level in the Environment Makes or Breaks the Second Law

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M. Hossein Partovi

Department of Physics & Astronomy, California State University, Sacramento

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Outline

Abstract

The extremes of correlation level in the environment are shown to lead to opposite directions of the thermodynamic arrow. In a low-entropy environment where the mean correlation level is low –the *Stosszahlansatz* scenario– the Clausius inequality is established using general arguments of quantum information theory. For a high-correlation environment –the *Verschränkung* scenario– a model of entangled systems is constructed in which heat flows from the cold to the hot body in direct violation of the second law. This model clearly shows that the second law of thermodynamics is a consequence of quantum statistical dynamics under conditions of low ambient correlations and breaks down under opposite conditions. When the *Stosszahlansatz* scenario prevails, the second law is valid for systems of any size, although its utility is limited for microscopic systems where fluctuations are not negligible relative to mean values. While the von Neumann entropy and other information-theoretic objects are used extensively in establishing the above results, all conclusions are based on energy flow between systems without any assumption on the thermodynamic interpretation of such objects. The results of this work strongly support the expectation, first expressed by Boltzmann and subsequently elaborated by others, that the second law is an emergent consequence of fundamental dynamics operating in a low-entropy cosmological environment functioning as an ideal information sink.

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- If our universe is indeed accelerating, we can count on the Earth to continue dumping waste information onto the surrounding space and maintaining a low-correlation environment where thermodynamics as we know it prevails.
- We use methods of quantum information dynamics and base our conclusions on energy flow thereby avoiding controversial issues of interpretation, e.g., of entropic quantities.

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Let the systems be labeled $i = 1, 2, \ldots N$, and the (von Neumann) entropies S^i. The two-body correlation information for the (i, j) pair is defined as $I^{ij} = S^i + S^j - S^{ij}$.

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Strong subadditivity implies $S^i + S^j \leq S^{ik} + S^{jk}$. There are $N(N-1)/2$ such inequalities, which, when aggregated, give

$$[N(N-1)/2]^{-1} \sum_{i<j=1}^{N(N-1)/2} I^{ij} \leq N^{-1} \sum_{i=1}^{N} S^i. \quad (1)$$

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The significance of this result is the **universal** validity of Boltzmann's ***Stosszahlansatz***, for all things large or small, as a likely condition in a low-entropy universe.

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We can use the non-negativity of the relative entropy $S(\rho_f \| \rho_i)$ to assert that

$$S(\rho_f \| \rho_i) = \beta_i \Delta U - \Delta S - \beta_i \mathrm{tr}(\rho_f \Delta H) \geq 0, \tag{2}$$

where $\Delta U = U_f - U_i = \mathrm{tr}(\rho_f H_f) - \mathrm{tr}(\rho_i H_i)$, and $\Delta S = S_f - S_i$. This is the fundamental inequality.

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But why? Does it, or the fundamental inequality, violate time-reversal invariance?

Let us repeat the above derivation, but this time for a process that starts from an arbitrary state and ends up in equilibrium. It is not difficult to see that instead of Eq. (2) one gets

$$S(\rho_i \| \rho_f) = -\beta_f \Delta U + \Delta S + \beta_f \mathrm{tr}(\rho_i \Delta H) \geq 0, \tag{6}$$

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Eqs. (5) and (7) are nevertheless consistent with equilibrium thermodynamics since, **for any small process whose endpoints are equilibrium states, both (7) and (5) apply, implying that** $dS = \delta Q / T$.

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For a *Stosszahlansatz* universe, Scenario S, Eq. (1) guarantees low correlation levels, so a typical pair of systems A and B, in equilibrium at temperatures T^A and T^B, will be initially uncorrelated. Therefore $\rho_i^{AB} = \rho_i^A \otimes \rho_i^B$ and $S_i^{AB} = S_i^A + S_i^B$.

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$$\Delta S^A + \Delta S^B \geq 0 \text{ (Scenario } \mathsf{S}\text{: systems initially in equilibrium)}. \quad (8)$$

This equation is a consequence of universal *Stosszahlansatz* and the origin of irreversibility in the second law.

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$$\sum_j Q_j^S/T_j^R \leq 0. \tag{9}$$

This is the Clausius inequality, equivalent to the second law.

Scenario V: breakdown of the second law

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$$|\Omega^{AB}\rangle = Z^{-1/2} \sum_i \exp(-\gamma\epsilon_i/2)|i; A\rangle|i; B\rangle, \tag{10}$$

where $|i; A\rangle$ ($|i; B\rangle$) is the ith energy eigenvector of system A (B), γ is a positive number, and $Z^{-1/2}$ is a normalization constant.

The individual states of the two systems, which are the marginal states of $|\Omega^{AB}\rangle\langle\Omega^{AB}|$, are easily found to be thermal equilibrium (Gibbs) states at temperatures $T^A = 1/(\gamma\mu^A)$ and $T^B = 1/(\gamma\mu^B)$, respectively.

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This reversal leads to $Q^A/T^A + Q^B/T^B \geq \Delta S^A + \Delta S^B$, which allows both directions of heat flow, including that **from the initially colder body to the hotter one**.

The individual states of the two systems, which are the marginal states of $|\Omega^{AB}\rangle\langle\Omega^{AB}|$, are easily found to be thermal equilibrium (Gibbs) states at temperatures $T^A = 1/(\gamma\mu^A)$ and $T^B = 1/(\gamma\mu^B)$, respectively. Once again we consider a process of heat exchange between A and B and find $Q^A \geq T^A \Delta S^A$, $Q^B \geq T^B \Delta S^B$, and $Q^A + Q^B = 0$. Contrary to Scenario S, here the joint state of the two systems is pure and ρ^A and ρ^B are isospectral, thus forcing the equality $S^A = S^B$ during the interaction and $\Delta S^A = \Delta S^B$ for the process. Since $Q^A Q^B \leq 0$, the above inequalities imply that $\Delta S^A = \Delta S^B \leq 0$, and

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How do we understand this bizarre behavior?

The clue is in the dual character of $|\Omega^{AB}\rangle$: (i) its purity forces S^A and S^B to move in lock-step, The clue is in the dual character of $|\Omega^{AB}\rangle$: (i) its purity forces S^A and S^B to move in lock-step, while, (ii) the initial equilibrium nature of its marginals implies that S^A and S^B can only decrease.

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Scenarios S and V are paradigms teaching us that the second law is not an independent or inviolable law of nature. Rather, it is a consequence of quantum mechanics conditioned on a low-correlation environment, and breaks down under the opposite conditions.

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- The reversibility issues that so haunted Boltzmann at the end of the nineteenth century are seen to arise from the initial conditions prevailing in the environment. To his credit, Boltzmann had already guessed that!
- I believe that a fundamental understanding of thermodynamic behavior incorporating the role of the environment is a prerequisite to the resolution of the reduction problem in quantum measurement theory.

This talk is for the most part based on

- M. Hossein Partovi, "Entanglement versus *Stosszahlansatz*: Disappearance of the thermodynamic arrow in a high-correlation environment," *Phys. Rev. E* **77**, 021110 (2008).
- M. Hossein Partovi, "Quantum Thermodynamics," *Phys. Letters* A **137**, 440 (1989).
- M. Hossein Partovi, "Irreversibility, reduction, and entropy increase in quantum measurements," *Phys. Letters* A **137**, 445 (1989).

Further results extending those of the first paper above can be found in

- David Jennings and Terry Rudolph, "Entanglement and the thermodynamic arrow of time," *Phys. Rev. E* **81**, 061130 (2010).

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