NAME:

Fall 2009 STAT 115A - Test III
DUE Tuesday 12/1/09 in class

PRESENT YOUR ANSWERS IN THE SPACE PROVIDED. YOU SHOULD GO OVER SECTIONS 3.3, 3.4 (EXCLUDE SECTION 3.4.1) AND THOROUGHLY UNDERSTAND $U[a,b], U(a,b), \Gamma(\alpha,\beta)$ AND ITS RELATIVES $-Exp(\lambda)$ (3.3.2, P.150), $\chi^2(r), \Gamma(k,(1/(\lambda))$ AS WAITING TIME W FOR k POISSON EVENTS (REMARK 3.3.1, P.150), $N(\mu,\sigma^2)$ (P.160 TO COROLLARY 3.4.1, P.1660, THE ENTIRE SECTION 2.2 AND SECTION 2.5. NOTE THAT EXAMPLE 3.3.6 AND THEOREM 3.4.1 USE THE CDF TECHNIQUE, THE TOP OF P.162 USES THE TRANSFORMATION TECHNIQUE, THEOREMS 3.2.1, 3.3.2 AND 3.4.2 USE THE MGF TECHNIQUE.

1. (8 POINTS) Let X be a continuous rv with the pdf

$$f(x) = \begin{cases} 3e^{-3x} & if \;\; x > 0 \\ 0 & if \;\; x \leq 0 \end{cases}$$

(a) Find the CDF $F(t)$.

(b) Find the mgf $M(t)$.

(c) Find the $E(X)$ and the $V(X)$.

2. (7 POINTS) Let $X \sim \beta(3,1)$.

   (a) Write down the pdf $f_X(x) =$

   (b) Let $Y = 3X$. Find the pdf $f_Y(y)$.

   (c) Let $U = Y^3$. Find the pdf $f_U(u)$ using the transformation technique.

3. (6 POINTS) (a) Let $\Theta \sim U\left(\frac{\pi}{2}, -\frac{\pi}{2}\right)$. Find the pdf $f(x)$ of $X = \tan\Theta$ using the transformation technique.

   (b) Do you know the name of this distribution?

4. (8 POINTS) Suppose the joint mgf of two rvs X_1, X_2 is given by

$$M(t_1, t_2) \;=\; (1 - 2t_1)^{-2}(1 - 3t_2)^{-1} \;,\; t_1 < (1/2), t_2 < (1/3).$$

(a) Show that the two rvs are independent.

(b) Find $E(X_1X_2)$.

(c) STATE the distributions of X_1 and X_2.

(d) Using your answers to parts (b) and (c) compute ρ.

5. (12 Points) Let X_1, X_2 have the joint pdf

$$f_{X_1,X_2}(x_1,x_2) = \begin{cases} 8x_1x_2 & if \;\; 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Let $Y_1 = u_1(X_1, X_2) = \frac{X_1}{X_2}$. Complete the following steps to find the distribution of Y_1. (Have you gone over Example 2.2.5?)

(a) $\mathcal{S} = \{ \qquad\qquad\qquad\qquad \}$

(b) Choose a suitable transformation $Y_2 = u_2(X_1, X_2) =$

(c) $\mathcal{T} = \{ \qquad\qquad ; \qquad\qquad \}$

(d) Have you made sure that $(u_1, u_2) : \mathcal{S} \longrightarrow \mathcal{T}$ is a $1-1$ transformation?

(e) The inverse transformations are

$x_1 =$

$x_2 =$

(e) The Jacobian $J =$

(f) The joint pdf $f_{Y_1,Y_2}(y_1, y_2) =$

(g) The marginal pdf $f_{Y_1}(y_1) =$

(h) The marginal pdf $f_{Y_2}(y_2) =$

(i) $Y_1 \sim$, $Y_2 \sim$

6. (4 POINTS) Find the Uniform distribution of the continuous type that has the same mean and the same variance as those of a χ^2-distribution with 24 degrees of freedom.

7. (10 POINTS) Fill in the blanks **appropriately:**

- $\Gamma\left(\frac{1}{2}\right)$ = ...
- $\Gamma\left(\frac{7}{2}\right)$ = ...
- If $X \sim U[a, b]$ with $E(X) = 3$ and $V(X) = 12$ then

  $a =$ ...., $b =$ ....
- $\int_0^\infty x^{(3/2)} e^{-(x/4)}\, dx=$ ....$\sqrt{\pi}$
- $\int_0^\infty x^{99} e^{-\frac{x}{2}}\, dx$ = ...
- $\int_0^1 x^{99} (1-x)^{49}\, dx$ = ...
- The relationship between the $Geo(p)$ distribution and the .... distribution is analogous to the relationship between the .... distribution and the $Gamma(r, \beta)$ distribution.
- The yield of grain on a plot of farmland and the length of a new-born child are often assumed to have .... distributions.
- If $f(x) = c4^{-x^2} x \in \Re$ is a normal pdf then $c =$ ....
- If Y has the Laplace distribution then the pdf of Y is

  $f_Y(y)$ =