MATH 1 – Dr. R's Handout for Chapters 1 & 2

YOU SHOULD READ THE HISTORICAL NOTES AND BIOGRAPHICAL SKETCHES ON THE MARGINS OF THE TEXT.

Read Chapter 1. Go over Number Patterns and Sum Formulas (pages 9 − 16) and Contributions of George Polya, Fibonacci, and De Morgan's year of birth (pages 18 - 22), Magic Squares (p.28), Charles Dodgson (p.36).

Read Chapter 2. Section 2.1 deals with the Definition of a set, three ways to designate a set and several examples. We define some Sets of Numbers:

$\mathbf{N}$ = the set of Natural Numbers = {1, 2, 3, ... }

$\mathbf{N}\cup\{0\}$ = the set of whole numbers = {0, 1, 2, 3, ... }

$\mathbf{Z}$ = the set of all integers = $\{.... -3, -2, -1, 0, 1, 2, 3, ...\}$

$n(A)$ = cardinality of A = number of elements in A

$\in$ means "is an element of" or "belongs to"

$\notin$ means "is not an element of" or "does not belong to"

CAUTION: In the listing of elements of a set each element must be represented only once.

∞ is the symbol to denote infinity (Who invented this sign?)

Practice problems for 2.1: 1−8, 9, 11, 13, 15, 17, 19, 33, 35, 37, 39, 41, 43, 45, 47, 49, 53, 55, 57, 59, 61, 63, 65, 67−84, 87−91.

Venn Diagrams are visual devices to understand Set Theory.

SET OPERATIONS:

Let A, B, C represent three sets which are subsets of a universal set U.

- $A = B$ if A and B have the same elements

- A' is the set of all elements not in A
  $= \{ x \mid x \notin A \}$

- $\emptyset =$ the empty set or the set that has no element. What is $n(\emptyset)$?

- A is a subset of B or $A \subseteq B$ if every element of A is also an element of B. We can also represent this by $A \supseteq B$.

- What does $A \nsubseteq B$ mean?

- I will abbreviate as mathematicians do the phrase "if and only if" by "iff".

- Notice now that we can say $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

- A is a proper subset of B if $A \subseteq B$ and $A \neq B$. In symbols $A \subset B$.

- Again $A \subset B$ iff $B \supset A$. Also notice that the 'relation' or 'ordering' $\subseteq$ for sets is similar to $\leq$ for numbers and that the 'relation' or 'ordering' $\subset$ for sets is similar to $<$ for numbers.

- So $\emptyset \subseteq B$ for every set B and $\emptyset \subset B$ for every nonempty set B.

- Let A be a set with $n(A) = n$. How many distinct subsets does A have? 2^n. In the class we described a proof of this result. $n(\emptyset) = 0$ but $n(\{\emptyset\}) = 1$. $\emptyset \subseteq B \subseteq U$ for all sets B.

- Read pages 50−64. Go over all the worked examples in the text.

  Practice problems for 2.2: 1−54, 67, 69, 70.

- Read the definitions of $A \cap B, A \cup B$ and Examples 1 , 2.

  Two sets A, B are called disjoint if $A \cap B = \emptyset$.

  The difference of sets A and B , written as $A - B$ is the set of all elements in A but not in B.

  $A - B = \{x | x \in A \;\; and \;\; x \notin B\}$

Is $A - B = B - A$?

In the ordered pair (a, b) where a is called the first component and b is called the second component.

In general, $(a, b) \neq (b, a)$.

We say that $(a, b) = (c, d)$ iff $a = c$ and $b = d$.

The Cartesian Product of sets A and B, written $A \times B$ is the set of all ordered pairs with first component from A and the second component from B.

What is $n(A \times B)$?

What are its properties?
How did the adjective Cartesian come about? What is a famous Latin quotation ascribed to René D. ? What does it translate as ?

Go over the examples 1 to 14 in section 2.3. Study the summary in page 62.

Use of Venn diagrams is discussed in detail on pages 62−64.

De Morgan's Laws (p.64) connect complementation with $\cap, \cup$.

(a) $(A')' = A$ (The complement of the complement of a set is the original set).

(b) $(A \cap B)' = A' \cup B')$ (The complement of the intersection of two sets is the union of the complements of the two sets).

(c) How do you state in words without using symbols the De Morgan's Law on the complement of the union of two sets? Write an equivalent expression in set theory using De Morgan's Laws for $(A' \cap B')'$.

- Practice problems for **2.3:** 1−6, Odd numbered exercises 7−79, 83, 87, 95, 109, 111, 115, 117, 119, 125.

We will go over lots of Practice problems in class. You must learn to do the rest.

- Section 2.4: Study the four worked examples. Example 4 deals with information in a tabular form. The cardinal number formula for $n(A \cup B)$ (page 69) is a special case of "counting using the inclusion exclusion principle".

- Practice problems for **2.4:** 1, 3, 5, 7, 9−16, 19, 21, 23, 25, 28, 31. TRY 34.

- Chapter 2 EXTENSION (pages 74 − 78) deals with Infinite sets and their cardinalities. Let us add to our collection of sets of numbers:

  $\mathbf{Q}$ = the set of rational numbers = $\{(p/q) | p, q \in \mathbf{Z}, q \neq 0\}$

  $\mathbf{R}$ = the set of real numbers = $\{x | x$ is a number that can be written as a (possibly infinite) decimal$\}$

  $\mathbf{A}$ = the set of algebraic numbers = the set of real numbers which are roots of polynomials with integer coefficients

  $\mathbf{I}$ = the set of irrational numbers = $\mathbf{R} - \mathbf{Q}$ = $\{x | x$ is a real number which cannot be written as a quotient of two integers with nonzero denominator $\}$

  Note that $\mathbf{A} \subset \mathbf{R}, \mathbf{I} \subset \mathbf{R}$ and $\mathbf{Q} \subset \mathbf{R}$, $\quad \mathbf{Q} \cap \mathbf{A} \neq \emptyset, \mathbf{A} \cap \mathbf{I} \neq \emptyset$. But $\mathbf{Q}$ and $\mathbf{I}$ are disjoint.

- The sets $\mathbf{N}$, $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{A}$, $\mathbf{I}$ are all examples of infinite sets. One way to think of a set as infinite is to say that its cardinal number is not in $\mathbf{N} \cup \{0\}$. However we want to define infinite sets by their set theoretic special property.

  We say that two sets A, B are *equivalent* (written as $A \sim B$) if we can match elements of A to elements of B in such a way that (i) every element $a \in A$ has a matching element $b \in B$ and (ii) any two distinct elements of A have distinct matching elements in B. Such a matching rule is called a *one−to−one correspondence.*

For finite sets A, B, C, D we have $A \sim B$ and if C is a proper subset of D then $C \not\sim D$.

For infinite sets C, D is it still true that if C is a proper subset of D then $C \not\sim D$? No. Why?

With this in mind, let us define a set to be infinite if it can be placed in one−to−one correspondence with a proper subset of itself.

Now we define $n(\mathbf{N}) = \aleph_0$, $n(\mathbf{R}) = c$. These concepts were introduced by Georg Cantor (Remember the title of his thesis?) who proved the intersting results: (i) $n(\mathbf{Q}) = \aleph_0$ and (ii) $\aleph_0 < c$. Study the table on page 78.

- Practice problems for EXTENSION EXERCISES: 1−10 , Odd numbered exercises from 11 to 40.

- Practice Chapter 2 Test.

- IT IS IMPORTANT THAT THE STUDENTS IN MY MATH 1 SECTIONS COMPLETE THE ASSIGNED PRACTICE PROBLEMS IN TIMELY MANNER AND MAINTAIN A FOLDER OF THE COMPLETED WORK. THE ODD NUMBERED EXERCISES HAVE THEIR SOLUTIONS AVAILABLE IN THE TEXT. FOR SOME OF THE ADVANCED PROBLEMS ASSIGNED, THE STUDENTS SHOULD CONSULT DURING THE OFFICE HOURS DR. R. AND THE TA AS WELL AS MATH LAB TUTORS DURING THE LAB HOURS TO ENSURE THAT THEY HAVE BEEN ABLE TO SOLVE THE PROBLEMS CORRECTLY. THIS IS THE BEST WAY TO PREPARE FOR THE MIDTERM TESTS.