## SPRING 2014, MATH 1 – Handout for Chapter 5

Chapter 5 deals with NUMBER THEORY. Read page 179. We are mainly concerned with $\mathbf{N} \cup \{0\}$ in this chapter. You should become conversant with

- $b \mid a$ *means* $b$ devides $a$,
- $b \not| a$ means $b$ does not divide $a$
- factor, divisor, multiple, factorization (p.180), prime number , composite number, Sieve of Eratothenes (p.181-182) (Go to `http://www.vex.net/~trebla/numbertheory/eratosthenes.html` to work on the sieve and note down the prime numbers between 2 and 400).
- Divisibility tests (Table 2 on p.182), and the FUNDAMENTAL THEOREM OF ARITHMETIC
- Obtaining the unique factorization by direct computation
- **Practice 5.1:** 1−24, Odd numbered exercises from 25 to 51
- The theorem on the infinitude of primes and its proof (p.186)
- The search for large primes, Mersenne numbers, GIMPS, Fermat Numbers, Euler formula, Escott formula
- **Practice 5.2:** 1−10, Odd numbered exercises from 11−27 , 54, 56, 58-60.
- In section 5.3 we study about Perfect numbers, Deficient and Abundant numbers, Amicable numbers, Weird and Dull numbers(?!), The man who knew infinity, Definition of CONJECTURE(p.2), Goldbach's conjecture, the Twin prime conjecture, Fermat's Last Theorem and Andrew Wiles
- **Practice 5.3:** 1−10, odd numbered exercises from 11 to 35, 39−41, odd numbered exercises from 51 to 61
- 5.4 treats the Greatest Common Factor(GCF), the Least Common Multiple(LCM), Relatively prime numbers, Three methods for finding the GCF and Three methods for finding the LCM

- **Practice 5.4:** 1−10, 13, 15, 17, 21, 23, 25, 27, 33, 35, 37, 41, 43, 45, 47, 49, 50, 65, 67, 69.

- In 5.1 we discussed the formula for the number of all divisors. If $n = p_1^a.p_2^b.p_3^c$ then $N$ has $(a+1)(b+1)(c+1)$ divisors. For example, $48 = 2^4.3$ and so 48 has $(4+1)(1+1) = (5)(2) = 10$ divisors.

- #12 , #13 (p.190): For $n = 42$ Euler's formula gives $(42)^2 - (42) + 41 = 1763 = 43.41$ which is composite. For $n = 43$ the formula gives $(43)^2 - (43) + 41 = 1847$ which can be verified to be a prime by checking for prime factors below 43 since 43 is close to $\sqrt{(1847)}$.

- You have to study 5.3 carefully to understand *proper divisors* and finding the sum of proper divisors. That can be used to classify every number as perfect, abundant or deficient. Why are all prime numbers deficient?

- 5.5 treats the well known Fibonacci Sequence $\{F_1, F_2, F_3, .....\}$ and the Golden Ratio $\phi = \frac{(1+\sqrt{5})}{2}$. Applications of these are discussed. **Practice 5.5:** 1-6, 15, 17, 20, 31.

- We will briefly discuss Modern Cryptography (p.205 − 211)

- As a review of your mastery of the concepts we discussed solve from Chapter 5 test: 1-16, 18, 19.

- Using a program called Mathematica (created by Stephen Wolfram) I could establish that **5.7.11.13.17.19.23.29.31 = 33,426,748,355** is an ABUNDANT number since the sum of its proper divisors is calculated to be **33,459,293,245**.