TERMINOLOGY FOR ARITHMETIC OPERATIONS IN ANCIENT GREEK

Geometry was the soul of ancient Greek mathematics. Archimedes and Apollonius gained fame with geometrical constructions; Euclid is still known to high-school students because of his exposition of geometry. From the second century B.C., however, there was a shift of interest from strictly geometrical problems to trigonometry, spherics, and numerical methods, i.e. the mathematics necessary for astronomy. {note} {This shift of interest noted by Knorr Problems 340.} These lines of investigation prompted by this shift of interest made ancient astronomy's precise description of the universe possible, precise not in its description of the universe's size and shape--which description was not in fact correct--but in its quite accurate predictions of solar, lunar, and planetary positions.

Now, ancient geometry is quite well known, ancient numerical methods (which I will hereafter call arithmetic){note} {note -In ancient texts "arithmetic" means number theory; I am discussing what was called "logistics."} less so. This paper will attempt to elucidate the basic style and terminology of ancient Greek arithmetic.[1] Further investigation would trace the undoubted ties between near eastern mathematics, as seen on the cuneiform tablets, and later Greek mathematics and astronomy, the use of numbers in daily life (as opposed, for example, to the use of the abacus), and the development of specialized methods in the professions, such as surveying, astronomy, mensuration, and so on.

With rare exceptions, no Greek mathematical writer used formulas or equations; most preferred to describe mathematical operations, even the simplest, in words.[2] This peculiarity alone, even ignoring the primarily geometrical orientation of ancient mathematical texts, makes the text of an ancient author read very differently from a modern mathematical text. What is today translated as a series of equations leading to a

solution was in fact written as a narrative paragraph.[3] In this narrative style of exposition, the particular sense of each word used for the various operations (and in this paper I will deal solely with the arithmetic operations: counting, addition, subtraction, multiplication, division) must be very clear. There are in fact more words in ancient Greek for these operations than there are symbols in modern notation (+ - x ÷), and each word is used in a slightly different sense. For example, as illustrated below, different words are used for 1) adding two independent numbers, 2) for adding a factor to a previously calculated number, and 3) for finding the grand total of a list of numbers. Moreover, different genres of mathematical writing adopt different sets of words: the usages of Euclidean geometry controlled the vocabulary of Theon of Smyrna (as it did the vocabulary of the philosophical commentators), while the usage of surveyors controlled Heron's. Vettius Valens is in a class by himself: the astronomers and astrologers, with their constant manipulations of charts and tables, had developed their own usages. In any event, this paper simply aims to list the terms commonly used for each of the operations mentioned above; to explicate their derivation and range of meaning, if any problems seem to arise from the use of the term; then to site the term in its mathematical context.

One question that may certainly arise: how many texts can there be which explicitly describe arithmetic operations, e.g. which tell in words how to add or multiply two quantities? There are in fact some: learners' manuals for astrology, astronomy, and mensuration, plus some texts of similar nature found among the Egyptian papyri.[4] These texts--Vettius Valens' Anthologiae and the voluminous works of Heron are the best examples extant--aim to instruct the learner in the mysteries of multiplication and division, the use of tables and charts. Theon of Alexandria's Commentary on the Almagest and his Short Commentary on the Handy Tables

perform a similar function for students of astronomy, Heron's Metrica for students of mensuration. My interest was piqued during a study of Vettius Valens' peculiar usages, and I hope to have elucidated obscurities in his text by comparisons with the usages of others.

I begin with the terms for **"to count"** including

ἀνατρέχω	ἀπολύω...καταλήγω/καταντάω
ἀριθμέω	ἐκβάλλω
ψηφίζω/ἐμψηφίζω	

ἀνατρέχω "to count back" as in the following passage from Vettius Valens 51.35-7K:

ταύτας ἐὰν ἀφέλω ἀπὸ τοῦ μεγίστου ὅρου τῶν σπη', λοιπαὶ σοη'· ταύτας ἀναδραμὼν ἀπὸ τῆς γεννη[μα]τικῆς ὥρας ψηφίσας τὴν Σελήνην εὑρήσεις ἐπὶ τοῦ κατὰ ἐκτροπὴν ὡροσκόπου.

"If I subtract this [10] from the maximum factor, 288, then 278 is left. If you count this amount back from the hour of birth and calculate the moon's position, you will find the Ascendant of the delivery."{footnote}

ἀπολύω "to count off" followed by καταλήγω "to stop counting." This usage seems to be confined to Vettius Valens; Theon of Alexandria uses ἀπολύω...καταντάω in exactly the same sense (Short Comm. 20.9).

Vettius Valens 310.3-5K:

...καὶ ἃς ἐπέχει ὁ Ἥλιος μοίρας κε' καὶ ἃς ἐπέχει ἡ Σελήνη ι', γίνονται νδ'. ταύτας ἀπολύω ἀπὸ τοῦ σεληνιακοῦ ζῳδίου καὶ καταλήγει Ταύρῳ μοίραις κδ', αἷς παράκειται Καρκίνος·

"...and the position of the sun, 25°, plus that of the moon, 10°, for a total of 54°. I count this amount off from the moon's sign and [the count] stops at Taurus 24°, at which position [in the chart] is Cancer."

ἀριθμέω has the sense "to discover what some unknown number is." Two examples illustrating this use: Vettius Valens 145.32-146.1:

Σκοπεῖν δεήσει, πότερον συνοδική ἐστιν ἡ γένεσις ἢ πανσεληνιακή, καὶ ἐὰν μὲν συνοδικὴ εὑρεθῇ, ἀριθμεῖν τὰς μοίρας ἀπὸ τῆς συνόδου ἐπὶ τὴν κατ' ἐκτροπὴν Σελήνην καὶ τὰ ἴσα ἀπολύειν ἀπὸ τοῦ ὡροσκόπου κατὰ τὸ ἑξῆς.
"It will be necessary to see if the nativity is at the full-moon or new-moon, and if it is found to be at the new-moon, count the degrees from the [position of the] new-moon to [the position of] the moon at the delivery; then count off an equal amount from the Ascendant in the direction of diurnal motion."
Note that in this example, ἀριθμέω is used for the count to determine the unknown number of degrees between the position of the new-moon which immediately preceded the infant's birth and the moon's position at the birth itself, while ἀπολύω refers to the counting off of this same--now known--number from a given point to a given point.

Vettius Valens 153.1-3:
...ἐπιγνόντας, ἐν ποίῳ ζῳδίῳ γέγονεν ἡ σύνοδος ἢ ἡ πανσέληνος, ἀριθμεῖν ἀπὸ τῆς αὐτῆς μοίρας κατὰ τὸ ἑξῆς ἕως ἀναβιβάζοντος...
"having determined in which sign the new- or full-moon occurred, count from that same degree, in the direction of proper motion, to the Ascending Node..." The astrologer needed to determine the distance between the sign and the Node; he is finding an unknown.

An example from Heron of Alexandria, de mensuris 24.1.4-6:
τὴν μείζω περιφέρειαν καὶ τὴν ἐλάσσω σύμμιξον· γίνονται πόδες ρμ· ὧν τὸ ἥμισυ· γίνονται πόδες ο. ἠριθμήσαμεν τὰ βάθρα τοῦ θεάτρου καὶ εὕραμεν ὄντα αὐτὰ ρ· πολυπλασίασον τοὺς ρ ἐπὶ τοὺς ο· γίνονται πόδες ζ· τοσούτους ἄνδρας χωρεῖ τὸ θέατρον, τουτέστιν ζ.
"Combine [=add] the greater and the lesser circumference; total 140. Take one-half; result 70. We measured [literally counted] the depth of the theater and we found it to be 100. Multiply 100 by 70; result 7000 feet. The theater holds that many men, i.e. 7000."

In Heron as well, ἀριθμέω is used for counting to determine an unknown.

ἐκβάλλω "to start, then continue, counting" (also διεκβάλλω in Vettius Valens). The basic mathematical sense of this word is "to extend, produce", as in the 23rd Definition of Euclid (Mugler 166; παρατείνω is also used in this sense):
Παράλληλοί εἰσιν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὖσαι καὶ ἐκβαλλόμεναι εἰς ἄπειρον ἐφ' ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπίπτουσιν ἀλλήλαις.
"Parallel lines are straight lines which, being in the same plane and being produced indefinitely in either direction, do not meet each other in either direction." The word is also used for producing a plane: Pappus, Collectio III 134.43ff and passim.
This usage was applied to counting in the sense "continue the count until you get to..." Vettius Valens 204.25-7K:
...μέριζε εἰς τὸν ιβ'· καὶ τὰ καταλειφθέντα ἔκβαλλε ἀπὸ τῆς Σελήνης τῆς κατὰ γένεσιν καὶ ἑκάστῳ ζῳδίῳ μίαν ψῆφον διδοὺς ἀπὸ τοῦ κεκληρωμένου ζῳδίου τὸν μῆνα διέκβαλλε. καὶ ὅπου δἂν καταντήσῃ ὁ ἀριθμός...
"...divide by 12. Count the remainder off from the moon's position at the nativity and giving 1° to each sign, count off from the sign which controls the month. Wherever the count stops..."

ψηφίζω "to count," money, votes, or other concrete objects, as in the following passage from Plutarch's Con. Praec. 141c:
Πάλιν ἡ Ὀλυμπιάς, αὐλικοῦ τινος νεανίσκου γήμαντος εὐπρεπῆ γυναῖκα κακῶς ἀκούουσαν, "οὗτος," εἶπεν, "οὐκ ἔχει λογισμόν· οὐ γὰρ ἂν τοῖς ὀφθαλμοῖς ἔγημε." δεῖ δὲ μὴ τοῖς ὄμμασι γαμεῖν μηδὲ τοῖς δακτύλοις, ὥσπερ ἔνιοι ψηφίσαντες πόσα φέρουσαν λαμβάνουσιν, οὐ κρίναντες πῶς συμβιωσομένην.
"Olympias [Alexander the Great's mother] said on hearing that a young fluteplayer had married a beautiful woman of bad reputation: 'This makes no sense; he didn't marry with his

eyes.' One must marry not with the eyes or the fingers--as some are said to, counting how much they will get in the dowry, but not calculating how they will live together."

P.Wisc. 2. 78 rp.r.4.132ff:
καὶ εἴ σοι δοκεῖ ψηφίσας ταῦτα ὅπως ἂν σύμβολα λάβωμεν· εἰ δὲ μὴ γράψας Ἰάσονι ὅπως ἂν τὰ σύμβολα ἡμῖν...
"... and if this is satisfactory to you, count this [sum] out so that we might get a receipt; if it isn't, write to Jason so that [he might get] the receipt for us."

"To count units of measure" in the following (a xesta=the Roman sextarius=a pint; modius=a peck):
Heron, Fragmenta 22.1.10-12:
...ἀνάλυσον τοὺς μοδίους εἰς ξέστας, καὶ ψήφισον τὸν μόδιον τοῦ σίτου κατὰ δ' ξέστας·
"...break the modius down into xestae and count the modius of grain as 4 xestae."

Occasionally ψηφιζω means "to calculate" as in Vettius Valens 53.10-12K (but even here Valens means "to count off entries in a table):
Λαμβάνω πάντοτε τὸν πρὸ τῆς γενέσεως ἐνιαυτὸν καὶ τὸν μῆνα τὸν γεννητικὸν καὶ τὴν ἡμέραν, καὶ ψηφίζω τὴν Σελήνην· καὶ ἐπιγνούς, ἐν ποίῳ ζῳδίῳ εὑρέθη, σημειοῦμαι.
"I take the year before the nativity and the birth month and the day [i.e. the birth date in the year before the actual birth] and I calculate the moon [=look up its position in a table]; I make a note of the sign in which it is found."

ἐμψηφιζω is found in a sense parallel to ἐκβάλλω "to count": ἐνψήφιζε δὲ τὰς μοίρας ἕως ἔλθῃς "Count off the degrees until you come to..." (P.Mich. 149 vii 6-7)

The results of counting or of the other operations are usually expressed by the following term:

γίνεται/γίνονται "equals"; this is a very common usage (Mugler 103); examples can be found under ἀπολύω above or προσλαμβάνω below. Sometimes the word occurs with εἰς: αἱ ἄρα κ′ μοῖραι γίνονται εἰς μυριάς α′ δ′. "Therefore 20° equals 14,000 stades" (Heron Dioptra 35.69).

I continue with the words for **"to add"** including

καί	συνάγω/ἐπισυνάγω
προσβάλλω	σύνολον
προσλαμβάνω	συντίθημι
προστίθημι	
συμμείγνυμι	

If a word for addition is not used, simple juxtaposition can indicate addition. (It can also indicate multiplication; see below.) There was no plus sign in Greek arithmetic.

καί "and" as in English "two and two are four." This usage does not seem common, perhaps because most surviving texts are not at such an elementary level. One, however, is: Würzburg K 1013 comprises a group of five wax tablets. Tablet 2, side B, and tablet 3, side B, contain simple addition practice:

	translation
η καὶ α θ	8 and 1, 9
α καὶ η θ	1 and 8, 9
θ καὶ β ια	9 and 2, 11
β καὶ θ ια	2 and 9, 11

and so on. The editor of this text suggests that the tablets come from a mixed Coptic-Greek milieu; perhaps this exercise was for writing, rather than arithmetic, practice; since the letters are also numbers, learning the alphabet also implied learning the digits.{}

προσβάλλω "to add a factor." In geometry this word means "to construct a line from a given point to another point/line": Euclid Data 94: διὰ τὲ τοῦ Δ πρὸς τὸν κύκλον προσβεβλήσθω τις τυχοῦσα ἡ ΔΑ - "Let a line DA be constructed through D to the circle."

See Mugler 364.
An example of the arithmetic meaning in Heron Geometrica 22.16:
Κύκλου τὴν περίμετρον εὑρεῖν. τὴν διάμετρον τριπλασίασον καὶ πρόσβαλε τὸ ζ′ τῆς διαμέτρου· καὶ ἔξεις τὴν περίμετρον.
"To find the perimeter of a circle: multiply the diameter by three and add 1/7 of the diameter. You will have the perimeter."

Two examples from Vettius Valens, 20.24-28 and 31.14-16:
Ἀπὸ τῆς κε′ τοῦ Ἐπιφὶ ἕως τῆς γεννητικῆς ἡμέρας ψήφισον τὰς ἡμέρας σὺν ταῖς ἐπαγομέναις καὶ πρόσβαλλε τῷ ἀριθμῷ μοίρας κβ′ καὶ ἀπόλυε τὸν ἀριθμὸν ἀνὰ λ′, ἡμέρας μὲν ἀπὸ Καρκίνου, νυκτὸς δὲ ἀπὸ Αἰγοκέρωτος.
"Count the days (including the intercalary days) from Epiphi 25 until the day of birth and add to this number 22°; count this number off by 30's, starting at Cancer for day births, at Capricorn for night births."

Εὗρον δὲ καὶ ἐν ποίῳ ζῳδίῳ ἡ Σελήνη ἀπὸ χειρὸς οὕτως. ἑκάστου βασιλέως τὴν πρόσθεσιν τῷ ζητουμένῳ ἔτει προσβάλλων μερίσεις εἰς τὸν γ′ μὴ λοιπογραφῶν τὸν ἀριθμὸν ἀλλὰ κατέχων·
"Handily find in which sign the moon is located thus: add the addition-factor for each emperor to the year in question; divide by 3, not discarding the remainder, but keeping it."{note}

In each of these examples, προσβάλλω refers, not to the adding of any two numbers during the course of a calculation, but to the addition of a specific adjustment factor. A parallel from trade and commerce:
P.Prag.Varcl 1.8:
Εἰρηναῖος Ἡρωνίνῳ τῷ ἀδελφῷ χαίρειν. παρέλαβον καὶ νῦν διὰ Ἑκύσεως ὀνηλ(άτου) [ἄ]λλα δίχω[ρ]α ὀκτώ, δί(χορα) 8. ἂν ἡδέως ἔχῃς, πρόσβαλε αὐτοῖς, ἐπεὶ χρῄζομεν αὐτῶν. ἐρρῶσθαί σε εὔχ(ομαι). 6 (ἔτους) Φαμενὼθ 28.
"Eirenaios to his brother Heronios greetings: I just received via Hekysis the muledriver another eight dichora (di. 8) [of wine]. if you please, send more in addition [=add to them],

since we need them. I pray that you are well. Year 6, Phamenoth 28."

προσλαμβάνω "to assign"; "to be added"Mugler 365; an example from Euclid Elements 13.1:
Ἐὰν εὐθεῖα γραμμὴ ἄκρον καὶ μέσον λόγον τμηθῇ, τὸ μεῖζον τμῆμα προσλαβὸν τὴν ἡμίσειαν τῆς ὅλης πενταπλάσιον δύναται τοῦ ἀπὸ τῆς ἡμισείας τετραγώνου.
"If a straight line is cut in extreme and mean ratio, the square on the greater segment, with one-half of the whole added to it, is equal to five times the square on the half."

Vettius Valens 34.17 (with corrected reading):
τὸν ἀριθμὸν δίπλωσον, καὶ ἑκάστου μηνὸς προσλαβὼν ἀνὰ β' ἥμισυ...
"Double the number and add 2 1/2 [=assign at the rate of 2 1/2] for each month..."

Diophantus Arith. 4.18 (ed. Tannery i.226.2ff):
Εὑρεῖν δύο ἀριθμούς, ὅπως ὁ ἀπὸ τοῦ πρώτου κύβος προσλαβὼν τὸν δεύτερον ποιῇ κύβον...
"To find two numbers such that the cube of the first added to the second shall make a cube..." (translation I. Thomas)
The parallel with Euclid may have influenced Diophantus to use this particular word here and in similar propositions.

προστίθημι "to add," one of the two customary words for ordinary addition, used for the addition of geometrical quantities and in number theory (Mugler 367). Contrast the uses of συντίθημι below.

The word as a technical term of mathematics in Plato R.P. 527a:
Λέγουσι μέν που μάλα γελοίως τε καὶ ἀναγκαίως· ὡς γὰρ πράττοντές τε καὶ πράξεως ἕνεκα πάντας τοὺς λόγους ποιούμενοι λέγουσιν τετραγωνίζειν τε καὶ παρατείνειν καὶ προστιθέναι καὶ πάντα οὕτω φθεγγόμενοι, τὸ δ' ἔστι που πᾶν τὸ μάθημα γνώσεως

ἕνεκα ἐπιτηδευόμενον.
"For they (geometers) speak in a most silly and restricted way, when in their teaching activities they claim to have some practical end in view--speaking of "squaring" and "producing" and "adding" and so on, whereas in fact the entire study is carried on for the sake of knowledge."

Theon of Smyrna de utilitate 33.5-10:
πρώτη μὲν ἡ μονάς·...τῆς δὲ ἑξῆς αὐτῇ δυάδος προστεθείσης γίνεται τρίγωνος ὁ γ'· εἶτα πρόσθες γ', γίνεται ς'· εἶτα πρόσθες δ', γίνονται ι'· εἶτα πρόσθες ε', γίνονται ιε'· εἶτα πρόσθες ς', γίνονται κα'·...
"First is the monad...When the succeeding duad is added to it, the triangle, 3, results. Next add 3, for a result of 6; next add 4, for a result of 10; next add 5, for a result of 15; next add 6, for a result of 21..."

Pappus, Collection III 78.1:
ὁμοίως δὲ κἄν αἱ Z,Γ δοθῶσιν, τὴν ὑπεροχὴν αὐτῶν προσθέντες τῇ Z τὴν γενομένην ἕξομεν ἴσην τῇ AB.
"Similarly, when Z and G are given, if their differences are added to Z, we will have the resulting [line] equal to AB."

Pappus, Collectio VIII 1066.27 (a section on calculating the mechanical advantage of gear trains):
ἐὰν ἄρα ἑνὶ αὐτῶν προσθῶμεν ὀλίγον τι βάρος, καταρρέψει καὶ ἐνεχθήσεται.
"If we add a small weight to one of the [sides], it will sink and incline."

συμμείγνυμι "to commingle" hence "to add", with reference to adding a group (more than 2) of previously given or calculated figures.

Vettius Valens 215.5-8K:
καὶ τὰς ἀπὸ τῆς γενέσεως ἕως τῆς ζητουμένης ἡμέρας συναγαγόντες καὶ ταῖς προτέραις ἐπισυμμίξαντες ἐκκρούομεν

δωδεκάδας, τὰς δὲ περιλειπομένας ἀπολύομεν ἀπὸ τοῦ ὡροσκόπου ἀνὰ μίαν ἑκάστῳ ζῳδίῳ διδόντες.
"Determine the days from the date of birth to the day in question and add this figure to the previous [figures]; divide by 12 and count off the remainder from the Ascendant, giving 1 to each sign."

Heron de mensuris 59.1.1-4:
Χώραν ἑτεροπλατοῦσαν ἐν τέσσαρσιν τόποις μετρήσωμεν οὕτως· ἔχει τὸ πλάτος ἀκαίνας $\overline{κ}$, τὸ δὲ παρὰ μέσον ἀκαίνας $\overline{ιε}$, ἔτι ἀκαίνας $\overline{ιβ}$, τὸ δὲ στενὸν ἀκαίνας $\overline{η}$· τὰ πάντα συμμίξας μέριζε τέταρτον· καὶ εὑρήσεις $\overline{ιγ}$ ἥμισυ δ′. τούτους πάλιν ἐπὶ τὸ μῆκος· καὶ εὑρήσεις τὴν ἀλήθειαν. ἡ ἄκαινα ἔχει πόδας $\overline{ιβ}$.
"We measure an area of breadth which varies in four places as follows: the width [at one end] is 20 akainai, in the middle is 15 akainai, at another place 12 akainai, the narrow end is 8 akainai. Add all these and divide by four; you will find it to be 13 1/2 1/4 [=13 3/4]. Multiply this by the length and you will discover the truth. (The akaina contains 12 feet.)"
A similar usage is found in de mensuris 24.1.4-6 quoted above, p. 00.

συνάγω or ἐπισυνάγω "to get a total by addition or a result by multiplication"; the participle refers to "the calculated result" (Mugler 395-6). Typically this word is not used for the operations of addition or multiplication, but in the sense "the result is..." equivalent to γίνεται (see above). The noun is ἐπισυναγωγή, although I have found this usage only in Theon of Alexandria (Short Comm. on the Handy Tables 208.9).

Vettius Valens 24.9-11:
ἐπεὶ ἐν τῷ πρώτῳ κλίματι ἀπὸ Καρκίνου ἕως Τοξότου συνάγονται ἀναφοραὶ σι′, τὸ ἕκτον γίνεται λε′· ἐν τούτοις ὁ Λέων ἀνενεχθήσεται.
"Since in the first klima there is a total of 210 rising times from Cancer to Sagittarius, one-sixth of this is 35. Leo will

rise in this [time]." Note that this passage refers to the simple addition of rising times, with the total of 210.

Vettius Valens 31.29-32.2:
ἐὰν γὰρ τὸ ἀπὸ Ἡλίου ἕως Σελήνης διάστημα πλεονάζῃ, ἐκ τούτων ἀφελεῖς τὸν πρῶτον συναχθέντα ἀριθμόν, καὶ αἱ καταλειπόμεναι τὴν ἡμέραν δηλώσουσιν.
"If [the figure for] the distance from the sun to the moon is greater, subtract it from the number that was calculated first, and the result will indicate the day."

Heron Alexandrinus, Liber Geeponicus 168.1-3 (on finding the area of a triangle):
Ἄλλως δὲ πάλιν. τὴν πλευρὰν ἐφ' ἑαυτήν, καὶ τῆς βάσεως τὸ ἥμισυ ἐφ' ἑαυτό· ὕφελε ἀπὸ τῶν συναχθέντων, καὶ τῶν καταλειφθέντων ποίει πλευρὰν τετραγωνικήν· ἔσται ἡ κάθετος.
"Another way [to find the area of an isosceles triangle]: [Multiply] the side by itself, then one-half of the base by itself; subtract [the latter] from the previously calculated number and take the square root of the result. This will be the altitude."

Theon of Alexandria, Short Commentary 213.13-214.2 (on converting seasonal hours to equinoctial [=clock] hours):
Καὶ τοὺς παρακειμένους αὐταῖς...ὡριαίους χρόνους ιε΄ ιζ΄ πολλαπλασίασαντες ἐπὶ τὰς ἀναδοθείσας καιρικὰς ὥρας ἀπὸ μεσημβρίας ε΄, τοὺς συναχθέντας χρόνους ος΄ κε΄ μερίσαντες παρὰ τοὺς τῆς μιᾶς ὥρας ἰσημερινῆς χρόνους ιε΄, τὰς γενομένας ἐκ τοῦ μερισμοῦ ὥρας ε΄ ι΄ ἔγγιστα ἔξομεν ὅσαι τότε γεγόνασιν ὧραι ἰσημεριναὶ αἱ δοθεῖσαι πέντε καιρικαί.
"We multiply the hourly 'times' entered [in the table at the sun's true position], i.e. 15;17, by the 5 seasonal hours after noon which we were given. We divide the resulting times, 76;25, by the 15 times of an equinoctial hour, and we have as the result of the division approximately 5 1/10 hours, which

are the equinoctial hours equivalent to the given five seasonal hours."

σύνολον "the total" of principal plus interest; P.Mich. 145 III vii 1.

συντίθημι or ἐπισυντίθημι "to combine," "to add"; the noun is σύνθεσις. This is the customary word for "addition" with reference to digits or numerical quantities (Mugler 403-4).

P.Mich. 144.3: σύνθες τὰ $\overline{\gamma}$ καὶ τὰ $\overline{\iota\eta}$ "add 3 and 18"

Vettius Valens 296.6-9:
καὶ ἐφεξῆς ἀνὰ ιβ′ προστιθέντες εὑρήσομεν τοῦ ἑξῆς ζῳδίου ταύτας παρακειμένας τῇ α′ μοίρᾳ· αἷς ἐπισυνθέντες τὰς ιβ′ καὶ λύοντες συνδέσμους τῇ τῶν ιδ′ μοιρῶν συνθέσει ὅλον τὸ ὄργανον συμπήξομεν.
"Adding 12 to each sign in order, we will find the [correct] figure to be entered next to the first degree of each sign. Then by adding 12 and breaking the sequence with the addition of 14°, we will construct the entire table."

Heron Metrica 1.6.16-19 (finding the area of a triangle when three sides are given; here 11, 13, 20):
...τὰ ιγ ἐφ' ἑαυτὰ γίγνεται ρξθ· καὶ τὰ ια ἐφ' ἑαυτά· γίγνεται ρκα· καὶ τὰ κ ἐφ' ἑαυτά· γίγνεται υ. σύνθες τὰ ρξθ καὶ τὰ ρκα· γίγνεται σϙ· ταῦτα ἄφελε ἀπὸ τῶν υ· λοιπὰ ρι. τούτων τὸ ἥμισυ· γίγνεται νε. παράβαλε παρὰ τὸν ια· γίγνεται ε. καὶ τὰ ιγ ἐφ'
"...13 times itself gives 169, and 11 times itself gives 121, and 20 times itself gives 400. Add 169 plus 121; result 290. Subtract this from 400; result 110. Take one-half of this; result 55. Divide by 11; result 5..."

In a school text--P.Vindob. G 26740, accompanied by a drawing:*
(*Cited in Harrauer-Sijepsteijn 169-70)
ἔστω μηνίσκος, οὗ ἡ μὲν ἐκτος περίμετρος σχοινίων $\overline{\tau}$, ἡ δὲ ἐντὸς σχοινίων $\overline{\iota\beta}$. βάσις σχοινίων $\overline{\beta}$. πόσων ἀρουρῶν ἔστιν;

ὡς δεῖ ποιῆσαι. συνθὲς τὰς δύο περιμέτρους, γείνεται κβ...
"Let there be a half-moon shape, whose outer perimeter is 10 schoinia, inner perimeter is 12, and base is 2. How many arouras is it? How one must do it: add the two perimeters; result 22..."

Theon of Smyrna de utilitate 27.7-10:
γεννῶνται δὲ διχῶς, ἔκ τε πολλαπλασιασμοῦ καὶ ἐπισυνθέσεως. ἐκ μὲν ἐπισυνθέσεως οἱ ἄρτιοι τοῖς ἐφεξῆς ἐπισυντιθέμενοι τοὺς ἀπογεννωμένους ποιοῦσιν ἑτερομήκεις.
"This [the production of rectagular numbers] happens in two ways, by multiplication and by addition. By addition, the even numbers, when added to the succeeding numbers, make the resulting numbers rectangular."

[summary about the words for adding.]
I continue with the words for "**to subtract**" including αἴρω, ἀφαιρέω, ἐκκρούω, λαμβάνω ἀπό, λοιπός, ὑφαιρέω. The most common is ἀφαιρέω. This word is also used in expressions meaning "to divide"; see below. The noun for the subtraction of geometrical quantities or digits is ἀφαίρεσις (Mugler 89-90).

Euclid, Common Notions 3:
Καὶ ἐὰν ἴσοις ἴσα προστεθῇ, τὰ ὅλα ἐστὶν ἴσα. Καὶ ἐὰν ἀπὸ ἴσων ἴσα ἀφαιρεθῇ, τὰ καταλειπόμενά ἐστιν ἴσα.
"If equals are added to equals, the wholes are equal. And if equals are subtracted from equals, the remainders are equal."

Vettius Valens 26.3-5
ἔλαβον ἀπὸ τῆς διαμετρούσης τὸν Ἥλιον μοίρας Λέοντος κβ' ἕως τῆς σεληνιακῆς· γίνονται οε'. εἰσὶν οὖν δωδεκατημόρια ς', ἅτινα ἀφαιρῶ ἀπὸ τῆς τοῦ Λέοντος μοίρας κβ'· λοιπαὶ ις', ἐν αἷς γέγονεν ἡ πανσέληνος.
"I took [the distance] from the degree in opposition to the sun--Leo 22°--to the moon's [position]; result 75°. This then is 6 dodecatemoria; I subtract this amount from Leo 22°. The

result is [Leo] 16°, the position of the [preceding] full moon."

Heron, Liber Geeponicus 82.1-7

Ἔστω κοῦπα καὶ ἐχέτω τὴν κάτω διάμετρον ποδῶν ε′, τὴν δὲ ἄνω ποδῶν γ′, τὸ δὲ ὕψος ποδῶν η′· καὶ ἐχέτω τὸν οἶνον ἕως ποδῶν ς′· πόσα οὖν κεράμια χωρήσει; ποιῶ οὕτως· ἀφαιρῶ τὰ γ′ ἀπὸ τῶν ε′· λοιπὸν β′· ταῦτα ἐπὶ τὰ ς′ γίνονται ιβ′· τούτων τὸ η′′ γίνονται α′ ἥμισυ· καὶ ἀφαιρῶ τὴν α′ ἥμισυ ἀπὸ τῶν ε′· λοιπὸν γ′ ἥμισυ· ἔσται οὖν τὸ πλάτος ἕως ὅπου ὁ οἶνος ἀνέβαινε ποδῶν γ′ ἥμισυ.

"Let there be a vat and let its lower diameter be 5 feet, its upper diameter 3 feet, and its height 8 feet. Let it contain wine to a depth of 6 feet. How many keramia will it hold? I calculate thus: I subtract 3 from 5; 2 is left. This figure times 6 gives 12. One-eighth of this gives 1 1/2. I also subtract this 1 1/2 from 5; 3 1/2 is left. Therefore the width to which the surface of the wine extends is 3 1/2 feet." (The calculation continues.)

Vettius Valens 22.23-5P

ἕκαστον δὲ ζῴδιον πόσην πρόσθεσιν ἢ ἀφαίρεσιν ἀναφορᾶς ἔχει, οὕτως γνωστέον. ἐπεὶ ὁ Κριὸς ἀναφέρεται ἐν κ′, ὁ Ζυγὸς ἐν μ′ εἰς συμπλήρωσιν τῶν ξ′...

"One must calculate thus to determine what the addition or subtraction factor for the rising time of each sign is: since Aries rises in 20 and Libra in 40 for a total of 60..."

αἴρω "to subtract," an uncommon usage based on the meaning "away with it/him", best illustrated from the Gospel of John 19:15:

ἐκραύγασαν οὖν ἐκεῖνοι, Ἆρον ἆρον, σταύρωσον αὐτόν.

"Then they shouted: 'Away, away; crucify him!'"

This word is also used meaning "to divide." See below.

Heron Geometrica 17.21.1-7:

Δοθείσης δὲ τῆς διαμέτρου τοῦ κύκλου μετὰ τῆς περιμέτρου

σχοινίων $\overline{νη}$ διαστεῖλαι καὶ εὑρεῖν, πόσου γίνεται ἡ διάμετρος καὶ πόσου ἡ περίμετρος. ποίει οὕτως· ἐὰν θέλῃς τὴν διάμετρον πρώτην εὑρεῖν, ποίησον τὰ $\overline{νη}$ ἑπτάκις· γίνονται $\overline{υς}$· τούτων λαβὲ μέρος $\overline{κθ}$· γίνονται $\overline{ιδ}$· τοσούτου ἡ διάμετρος. ταῦτα ἆρον ἀπὸ τῶν $\overline{νη}$· λοιπὰ $\overline{μδ}$· τοσούτου ἡ περίμετρος.

"Given that the diameter plus the perimeter of a circle is 58 schoinia, divide [this amount] and find how great the diameter is and how great the perimeter. Do it thus: if you wish to find the diameter first, make 58 seven times; the result is 406. Take 1/29 of this; the result is 14. That is the diameter. Subtract this from the 58 [total]; the remainder is 44, the perimeter."

ἐκκρούω "to subtract" as well as "to divide":

Pappus, Collectio VII 946.16-17:
κοινὸν ἐκκεκρούσθω τὸ ἀπὸ BZ
"Let BZ^2 be subtracted from both [geometrical figures/sides of the equation]"
See below under "to divide" for other senses of this word.

ὑφαιρέω "to subtract" used particularly by Heron of Alexandria.:

Heron Geometrica 12.38.3-6
ταῦτα μερίζω ἐπὶ τὰ $\overline{θ}$ τῆς βάσεως· γίνονται $\overline{κα}$· προστιθῶ τὰ $\overline{θ}$ τῆς βάσεως· γίνονται $\overline{λ}$· ὧν τὸ ἥμισυ $\overline{ιε}$. ἀπὸ τούτων ὑφαιρῶ τὰ $\overline{θ}$ τῆς βάσεως· λοιπὰ $\overline{ς}$·
"I divide this [189] by the 9 [=dimension] of the base; result 21. I add the 9 of the base; result 30. One-half of this is 15. From this I subtract the 9 of the base; remainder 6."

λαμβάνω ἀπό seems **not** to mean "to subtract." The examples I have found of this phrase in mathematical authors mean "to take the distance/amount between x and y" or "to take the next number in order":

Vettius Valens 36.24-27K
Ὁμοίως ἐπὶ τῆς αὐτῆς γενέσεως καὶ τὸν Ἑρμῆν οὕτως ἐψήφισα· ἔλαβον τὰς ἀπὸ Θὼθ ἕως τῆς ιγ′ τοῦ Μεχὶρ ρξγ′ καὶ ἔξωθεν προσέθηκα ρξβ′, ὁμοῦ τκε′· ταύτας ἀπέλυσα ἀπὸ Κριοῦ ἀνὰ λ′, κατέληξεν κε′ Ὑδροχόου· ἔνθα ὁ ἀστήρ.
"Also for the same nativity I calculated Mercury as well in the following way: I took the [days] from Thoth [1] until Mechir 13--this is 163--and I added as the external factor 162, for a total of 325. I counted this off from Aries, giving 30 [to each sign] and stopped at Aquarius 25°. The star [Mercury] was there."

Theon of Smyrna De utilitate 25.11-13
ὁποῖοί εἰσιν ὁ λβ′ ξδ′ ρκη′ καὶ οἱ ἀπὸ τούτων ἑξῆς κατὰ τὸ διπλάσιον λαμβανόμενοι. τὰ γὰρ λβ′ γέγονε μὲν ἔκ τε δ′ καὶ η′, ἃ ἐστιν ἄρτια·
"Of a similar nature are 32, 64, 128, and those numbers following these derived by doubling [the previous number]. For 32 derives from 4 and 8, which are even."

This use of λαμβάνω ἀπό casts doubt on the reconstruction of P.Mich. 145 III iv.12. where the editors print [ελα]βο[ν ι απο των] (δραχμων) κ· εγ[ε]γ[ετο ι· και το] ι′...
Which is supposed to mean: "I took 10 from the 20 drachmas; result 10; and one-tenth..."
I suggest that some other word must be hidden here, especially considering that ἔλαβον is used throughout the passage for "to take a fraction of some quantity": [ελα]βον το κ′ τω[ν λο]ιπων (δραχμων) φ | κδ, λοιπ(αι) κ· "I took 1/20 of the remaining drachmas (500); result 24, remainder 20."

λοιπός and its cognates are used for the results of a subtraction, "the remainder" (Mugler 276-7). The same word is used for the remainder of a division; for examples of this latter usage, see below.

Theon of Alexandria Commentary on the Almagest 1.10 (Loeb I 52):

ἀφαιροῦμεν ἑπτάκις τὰ ιβ′ πρῶτα ἑξηκοστά, τουτέστιν πδ′ δεύτερα ἑξηκοστά...καὶ ὑπολείπεται λοιπὰ ωλα′ δεύτερα ἑξηκοστά.

"We subtract seven times 12', which is 84''...and there is left a remainder 831'' (translation I. Thomas).

Now the terms for **"to multiply"** including the definite article, πολλαπλασιάζω, ἐπί, συνάγω, τίθημι. Multiplication seems to be considered as a series of additions (see Euclid Elements 7, def. 15 quoted below), just as division is considered as a series of subtractions. Hence simple juxtaposition can indicate multiplication, as well as addition. The wax tablets cited under καί above also contain multiplication tables which juxtapose the two numbers to be multiplied:{note}
{Würzburg K 1014 in Brashear, "Holz- und Wachstafeln" 19. Parallel passages cited in Brashear p. 19.}

	translation
ξ α ξ	60, 1, 60
ξ β ρκ	60, 2, 120
ξ γ ρπ	60, 3, 180
ξ δ Σμ	60, 4, 240
ξ ε τ	60, 5, 300
ξ ς τξ	60, 6, 360

and so on.
One might compare this set up with the usual fraction tables, customarily arranged as follows:{note}
{P.Mich. 146 vii (partial). The second entry in this sort of table is always 6000 times the fraction; this is included because 6000 drachmas make a talent. Using this entry, one can readily see how many drachmas make a given fraction of a talent.}

	translation
της α το ι΄ι΄	the 1/10 of 1, 1/10
το ι χ΄	the 1/10 [of 6000], 600
των β ε΄	of 2, 1/5
των γ ε΄ι΄	of 3, 1/5 1/10
των δ γ΄ι΄ε΄	of 4, 1/3 1/15

τῶν/τοῦ/τῆς More commonly than juxtaposition alone, the genitive case of the article is used to signal multiplication by a fraction, as in the immediately preceding example (τῆς, τῶν). Other examples, the first with λαμβάνω plus genitive:

λαβέ τὸ ς΄ τοῦ δευτέρου "Take one-sixth of the second number." P.Mich. 144 ii.2 or τούτων τὸ $\overline{\rho}$ $\overline{\rho\mu\zeta}$ ... "one-hundredth of this = 147..." P.Lond. ii 265.28. Note in both examples the article (τό) with the fraction and the genitive case of the multiplicand, as in the first line of the fraction table quoted above.

A similar use in Heron Metrica·6.1.8.4-6:
σύνθες τὰ ζ΄ καὶ τὰ η΄ καὶ τὰ θ΄· γίγνεται κδ΄. τούτων λαβὲ τὸ ἥμισυ· γίγνεται ιβ΄. ἄφελε τὰς ζ΄ μονάδας· λοιπαὶ ε΄.
"Add 7 and 8 and 9. Total 24. Take one-half of this; total 12. Subtract the 7 units; remainder 5."

πολλαπλασιάζω and πολυπλασιάζω (in later texts) are the most common words for "to multiply"; πολλαπλασιασμός is the noun meaning "multiplication"; πολλαπλάσιος means "a multiple".

Euclid Elements 7, Definition 15
Ἀριθμὸς ἀριθμὸν πολλαπλασιάζειν λέγεται, ὅταν, ὅσαι εἰσὶν ἐν αὐτῷ μονάδες, τοσαυτάκις συντεθῇ ὁ πολλαπλασιαζόμενος, καὶ γένηταί τις.
"A number is said to multiply a number whenever the number that is multiplied is added to itself as many times as there

are units in [the multiplier], and thus some number is produced."

Vettius Valens 35.18-19
τὰς δὲ περιλειφθείσας ιη′ πολυπλασιάζω ἐπὶ τὸν ιβ′, γίνονται σις′·
"I multiply the remainder, 18, by 12, with the result 216."

Heron Geometrica 12.38.1-2
Ἄλλως τὸ αὐτὸ τρίγωνον ἀμβλυγώνιον. πολυπλασιάζω τὰ ιζ ἐφ' ἑαυτά· γίνονται σπθ· ἀπὸ τούτων ὑφαιρῶ τὰ ι ἐφ' ἑαυτὰ γενόμενα ρ· λοιπὰ ρπθ.
"Another method with the same obtuse triangle: I multiply 17 by itself; result 289. I subtract from this the 100 (which came from 10 times itself); remainder 189."

Theon of Smyrna, De utilitate 10-12
καὶ γὰρ ἅπαξ ἓν ἕν, καὶ μέχρις ἀπείρου ἐὰν πολλαπλασιάζωμεν τὴν μονάδα, μένει μονάς.
"For one times one is one, and even if we multiply the unit to infinity, it remains a unit."

ἐπί Another very common way of expressing "to multiply" is the use of the proposition ἐπί, occasionally with the verb ποιέω, but often by itself (Mugler 187-190).

P.Mich. 145 III iii.2
ἐπόησα τὰ φόρτ(ια) ρ ἐπί τας (δραχμας) λ | Γ·
"I multiplied the 100 loads by 30 drachmas; result 3000."
(The vertical line in the Greek text is an abbreviation for γίνεται.)

Heron Stereometrica 1.5.1.1-4
Ἄλλως μετρῆσαι τὴν ἐπιφάνειαν. ποίησον οὕτως· τὴν διάμετρον ἐφ' ἑαυτήν· γίνονται ρ. ταῦτα ποίησον ἐπὶ τὰ μδ· γίνονται ͵Δυ. τούτων λαβὲ τὸ ιδ′· γίνονται τιδ δ′ κη′.
"Another way to measure the surface. Do thus: the diameter

times itself; result 100. Multiply this times 44; results 4400. Take 1/14 of this; result 314 1/4 1/28 [=314 5/28].

Eutocius Comm. on Archimedes Measurement of a Circle (Archim. ed. Heiberg III 242) (Loeb I 48)
ρνγ ἐπὶ ρνγ
"153 times 153"

συνάγω "to get a result by multiplication"; see above p. 00.

τίθημι ἐπί occasionally seems to mean "to multiply":

Heron Geometrica 24.10.15-20
τοῦτό ἐστιν ἡ κάθετος, ποδῶν κ. καὶ θὲς πάλιν τὰ μα καὶ πρόσθες α· γίνονται πόδες μβ· ὧν ἥμισυ γίνεται πόδες κα· ἔστω ἡ βάσις ποδῶν κα. καὶ θὲς τὰ λε καὶ ἆρον τὰ ς λοιπὸν μένουσι πόδες κθ. ἄρτι θὲς τὴν κάθετον ἐπὶ τὴν βάσιν· ὧν ἥμισυ γίνεται πόδες σι·
"This is the perpendicular, 20 feet; Take the 41 and add 2: result 42 feet. One-half of this is 21 feet. Let the base be 21 feet. Take the 35 and subtract 6; there remain 29 feet. Now multiply the perpendicular and the base; [result 420], one-half of which is 210."
In geometry τίθημι ἐπί means "to situate or construct a figure on a given place" (Mugler 421). In this passage θές really means "set" the perpendicular on the base. Hence the choice of this particular expression.

Words meaning **"to divide"** and the related nouns, including

αἴρω	μερίζω παρά
ἀφαιρέω	παρά
διαιρέω	παραβάλλω
διανομή	τέμνομαι
ἐκκρούω ἀνά	

Division was viewed as a series of subtractions, hence the vocabularies of the two operations overlap. It can also be

viewed as multiplication by a fraction (=division by its reciprocal). For the latter see under τῶν/τοῦ/τῆς above.

ἀφαιρέω means "to subtract" (see above) and "to divide", as does αἴρω.

Vettius Valens 21.31-22.1K
ποιήσομεν καὶ τὸν τῆς Σελήνης γνώμονα οὕτως. διπλώσαντες ἃς ἔχει μοίρας ἡ Σελήνη ἀφαιροῦμεν τριακοντάδας, τὰς δὲ περιλειπομένας δωδεκαπλασιάσαντες καὶ ἐπιπροσθέντες τὰς τῆς Σελήνης [ἢ] μοίρας ἀφαιροῦμεν τριακοντάδας, καὶ αἱ λοιπαὶ ἔσονται τῆς Σελήνης γνώμων.
"We will make the lunar gnomon as follows: we double the degree position of the moon and divide by 30 [=subtract 30's]. We multiply the remainder by 12 and add the degrees of the moon, then divide by 30. The remainding figure will be the lunar gnomon."

αἴρω meaning "to divide":

P.Rylands 27.1-2.
τὰ πλήρη ἔτη, πρ(όσθες) β΄, ἆρον ἀνὰ κε΄, τὰ λοιπὰ ἐπὶ τξε΄
"[Take] the full years, add 2, divide by 25 [literally "subtract by 25's"], [multiply] the remainder by 365..."{note}
{For a discussion of this astronomical fragment see O. Neugebauer, "The Astronomical Treatise P. Ryl. 27" Danske Vid. Selsk. 32.2(1949), B.L. van der Waerden "The Astronomical Papyrus Rylands 27" Centaurus 5(1958) 177-191, and A. Jones, "248-Day Schemes". For ἀνά here, compare the passage from Vettius Valens 33.19P quoted above; it means "at the rate of" in commercial language.}

διαιρέω "to subdivide": this word does not commonly refer to the arithmetic operation, but to the process of "splitting" a quantity in a given ratio or of "graduating" a length. The noun is διαίρεσις (Mugler 130-1). A synonym is τέμνομαι. The first two examples below illustrate the "graduate" sense.

Ptolemy Syntaxis I 10 (Heiberg i. 31.11-16):
τὴν μὲν περίμετρον εἰς τξ τμήματα διελόντες...τῆς διαμέτρου διὰ τὸ ἐξ αὐτῶν τῶν ἐπιλογισμῶν φανησόμενον ἐν τοῖς ἀριθμοῖς εὔχρηστον εἰς ρκ τμήματα διῃρημένης.
"We subdivide the perimeter into 360 segments...with the diameter subdivided into 120 segments because of the obvious convenience [of this number] for the numerical calculations."

Eudoxus Fragmenta, (ed. F. Lasserre) 80.3-5:{note}
{Quoted by Hipparchus in In Arati et Eudoxi Phaenomena Commentariorum I 6.4, ed. Manitius, Leipzig (1894).}
τῆς Μικρᾶς Ἄρκτου· ταύτης γὰρ ὁ ἔσχατος καὶ λαμπρότατος ἀστὴρ κεῖται κατὰ τὴν ιη′ μοῖραν τῶν Ἰχθύων, ὡς δὲ Εὔδοξος διαιρεῖ τὸν ζῳδιακὸν κύκλον, κατὰ τὴν γ′ μοῖραν τοῦ Κριοῦ.
"...the Little Bear: the last and brightest star of [this constellation] lies at Pisces 18°--or, as Eudoxus subdivided the zodiacal circle, at Aries 3°."

In Euclid and in the more geometrically minded authors the word is used in a geometrical sense: "to split a quantity/line in a certain ratio."
Euclid, Elements 10.45:
Ἡ μείζων κατὰ τὸ αὐτὸ μόνον σημεῖον διαιρεῖται.
"A major [straight line] is divided at one and the same point."

Theon of Smyrna De utilitate 18.15-18:
...ἀμέριστον καὶ ἀδιαίρετον τὸ ἓν ὡς ἕν. καὶ γὰρ ὁ μὲν ἄλλος ἀριθμὸς διαιρούμενος ἐλαττοῦται καὶ διαιρεῖται εἰς ἐλάττονα αὑτοῦ μόρια, οἷον τὰ ς′ εἰς τὰ γ′ καὶ γ′ ἢ δ′ καὶ β′ ἢ ε′ καὶ α′.
"The One, as One, is indivisible and undivided. For another number, when split, is lessened and subdivided into parts smaller than itself, e.g. 6 into 3 and 3, or into 4 and 2, or into 5 and 1."

But again the word may be applied to a numerical value, in a sense not too different from the "graduate" sense found in

Ptolemy, Syntaxis 1.10 above:

Heron Geometrica 4.7.5-15 (in a passage describing Roman units of measure):

καὶ τὸ μὲν ἰούγερόν ἐστιν ἐμβαδῶν ποδῶν $\overline{\beta}$ $\overline{\eta\omega}$· ἔχει γὰρ μῆκος ποδῶν $\overline{\sigma\mu}$, πλάτος ποδῶν $\overline{\rho\kappa}$· διαιρεῖται δὲ εἰς οὐγκίας $\overline{\iota\beta}$, ὡς εἶναι ἑκάστην οὐγκίαν ποδῶν $\overline{\beta\upsilon}$. καὶ αὐτὴ δὲ ἡ οὐγκία διαιρεῖται εἰς σκρίπουλα ἤτοι γράμματα $\overline{\kappa\delta}$, ὡς εἶναι ἕκαστον σκρίπουλον ποδῶν $\overline{\rho}$.

"The iugerum is 28800 square feet, for it has a length of 240 feet and a width of 120 feet. It is divided into 12 unciae, so that each uncia is 2400 feet. The uncia itself is divided into 24 scripula, or lines, so that each scripulum is 100 feet."

διανομαί "distributions" means something like "division" in Plato, e.g. Laws 819b4: μήλων τέ τινων διανομαὶ καὶ στεφάνων "the distributions of apples and crowns." (These are the useful parts of mathematics which the children should learn in play.) The word is not used for the arithmetical operation, solely for the distribution of things or of land, as in the the proemion to Heron's Metrica, where he recounts the origin of mensuration: "the first geometry, as ancient history tells us, was engaged in the measurement and distribution of land" (...περὶ τὰς ἐν τῇ γῇ μετρήσεις καὶ διανομάς). Heron mentions this "fact" several times in his work, as does Herodotus (Histories 2.109: the Pharaoh Sestostris developed geometry to measure the fields after the Nile had receded), and it has been widely repeated.

ἐκκρούω ἀνά "to divide by" literally "to cast out" followed by the number which is to be cast out. (See above under "to subtract".) The usual meaning of the word is "to push out" as in the following example.

Heron Pneumatica 1.proem. 52-54:

αἰσθήσεται τὸ πνεῦμα ἐκπῖπτον ἐκ τοῦ ἀγγείου· τοῦτο δὲ οὐκ

ἄλλο τί ἐστιν ἢ ὁ ἐκκρουόμενος ὑπὸ τοῦ ὕδατος ἀήρ.
"The wind/breath is felt coming out of the jar. This is nothing other than the air pushed out by the water."

It is a common word for "to divide" in Vettius Valens.

Vettius Valens 27.20-25K:
Ἐμοὶ δὲ μᾶλλον φυσικώτερον ἔδοξε τὰ πλήρη [τὰ] ἀπὸ Αὐγούστου ἔτη καὶ τὰς ἐμβολίμους λαβόντα, καθὼς πρόκειται, καὶ τὰς ἀπὸ Θὼθ ἕως τῆς γενεσιακῆς ἡμέρας καὶ ἐκκρούσαντα ἑβδομάδας καὶ τὰς λοιπὰς ἀπολύσαντα ἀπὸ Ἡλίου ἐκεῖνον κρίνειν τοῦ ἔτους κύριον, εἰς ὃν κατέληξεν ὁ ἀριθμός·
"To me it seems more scientific to take the full years from Augustus plus the intercalary years, as was just stated plus the days from Thoth [1] to the birth date; then divide by seven [literally "cast out 7's] and count the remainder from the sun's [day]. Consider that [day's star], where the count stops, to be the ruler of the year."

Vettius Valens 34.12-14K:
Τὸν δὲ [Ἄρεα] οὕτως. Τὰ ἀπὸ Αὐγούστου μέχρις <οὗ> ζητεῖς ἔτους ἀναλαβὼν ἔκκρουε ὁσάκις δύνῃ ἀνὰ λ', τὸν δὲ καταλειπόμενον ἀριθμὸν διάκρινε, πότερον ἄρτιός ἐστιν ἢ ἄνισος...
"Mars thus: take the years from Augustus to the year which you are investigating; divide by as many 30's as possible; note the remaining number, whether it is odd or even..."

μερίζω παρά "divide by" defined explicitly as
εὑρεῖν ποσάκις ἐστὶν ὁ κε ιβ ι ἐν τῷ αφιε κ ιε
"to discover how many times 25° 12' 10'' is in 1515°" Theon of Alexandria, Comm. on Ptolemy's Almagest I 10. This is perhaps the most common word for "to divide." The noun is μερισμός="division" and μέρος="fraction with numerator one (1)".

Vettius Valens 34.26-28K:
Τὴν δὲ Ἀφροδίτην οὕτως. τὰ ἀπὸ Καίσαρος ἕως οὗ ζητεῖς ἔτους

ἀναλαβὼν μέριζε παρὰ τὸν η′, τὸν δὲ καταλειφθέντα ἐντὸς τῶν η′ ἀριθμὸν σκόπει εἰ ἔχει ἐποχὴν στηριγμοῦ...
"Venus thus: take the years from Caesar until the year you are investigating; divide by 8; examine the remainder--less than 8--to see if it is at a stationary point.

Vettius Valens 35.31-33K:
Τὴν δὲ Ἀφροδίτην οὕτως. ρλθ′ ἐμέρισα παρὰ τὸν η′, λοιπαὶ γ′, αἳ σημαίνουσι στηριγμόν, ἐποχὴν δὲ Φαωθὶ ι′ Τοξότῃ.
"Venus thus: I divided 139 [years since Caesar] by 8; remainder 3; this indicates a point of maximum elongation in Sagittarius at Phaophi 10."

Heron Geometrica 24.5.7-8:
καὶ τὰ $\overline{ρπ}$ μερίζω παρὰ τὸν $\overline{ε}$, καὶ $\overline{λς}$ ἐστιν...
"I divide the 180 by 5, and it is 36..."

διαμερίζω is used for dividing a quantity of money in P.Genova 2 App. 1 rp. 4-9 (a partnership agreement):
ὁμολογῶ ἔσχον παρὰ σοῦ Πουῶνσις ἀργυρίου δραχμὰς ἐν ταλάντοις [τα]λαντων δώδεκα, γ(ίνονται) (τάλαντα) 12, εἰς λόγον τ[ῆς αὐτῆς ἐνπορίας] ἵνα εἴ τι εὕρωμεν ἐπάνω τῶν δώδεκα 12 [ταλάντων.....ερων διαμερίζομεν αὐτὰ ἐν δύο 2 μέρη, [ἐμοὶ μὲν ἥμισυ μέρος] σοὶ δὲ τὸ ἄλλον μέρος ὑπὲρ τῶν ἀργυ[ρίων·
"...I agree that I have had from you, Pouonsis, silver drachmas amounting to twelve talents (= tal. 12) for the account of this same enterprise and that if we find some profit beyond the twelve (12) talents...we will divide it into two (2) parts, one-half part to me, the other part to you, of the money..."

παρά by itself in the sense "divided by" in Würzburg K1024:
τὸ β′...
τῶν γ β γ β παρ(ὰ) γ βγ
"2/3...
of 3 = 2 3·2 [=6] divided by 3. 2·3[=6]." {note}
{This is a fraction table like the one cited above under the

beginning of the multiplication section. The editor, Brashear, considers that the right part of each line is a check on the accuracy of the multiplication by 2/3. W. Brashear, "Neue griechische Bruchzahlentabellen" Enchoria 12(1984) 1-6.)

παραβάλλω παρά "to divide;" used with a geometrical figure in mind, since this is the term for "applying" one area to another in geometry (Mugler 324). Consequently this term is not normally used for ordinary numerical division. (note)
(in the 13th century George Pachymeres, in a long section showing how to multiply and divide sexagesimal numbers, used παραβάλλω for "to divide" but felt the need to gloss the word when using it for this strictly numerical work.
Quadrivium 337.19-21:
Ἔπειτα δέον εἰπεῖν καὶ πῶς δεῖ παραβάλλειν εἴτουν μερίζειν αὐτοὺς παρ' ἀλλήλους εἰς τὸ εὑρίσκειν ποῖος ἀριθμός ἐστιν ὁ ὑπὸ τῆς παραβολῆς αὐτῶν γινόμενος, ὃν οἱ γεωμέτραι πλάτος καλεῖν εἰώθασιν.
"Next it is necessary to say how one must apply, i.e. divide, one number by another in order to discover what number is the result of the division; the geometers call this number the platos" [literally, the width]. The word from the geometric and philosophic tradition (for the close connection between these see Knorr Problems 367) is here applied to arithmetic--with an explanation.)
The underlying meaning can be seen in the following quote, the lemma to Euclid Elements 10.16:

Ἐὰν παρά τινα εὐθεῖαν παραβληθῇ παραλληλόγραμμον ἐλλεῖπον εἴδει τετραγώνῳ, τὸ παραβληθὲν ἴσον ἐστὶ τῷ ὑπὸ τῶν ἐκ τῆς παραβολῆς γενομένων τμημάτων τῆς εὐθείας.
"If to any straight line there be applied a parallelogram deficient by a square figure, the applied parallelogram is equal to the rectangle contained by the segments of the straight line resulting from the application." (translation Heath)

Theon of Alexandria Comm. on Ptolemy's Almagest 1.10:
παρὰ τὰ γενόμενα $\overline{ρλδ}$ παραβαλοῦμεν τὰ $\overline{χξ}$ ἑξηκοστὰ πρῶτα καὶ τῶν γενομένων ἐκ τῆς παραβολῆς $\overline{δ}$ πρώτων ἑξηκοστῶν ἕξομεν...
"We divide the resulting 134 into 660′ and from the division we will get 4′..."
This passage refers to an illustrative figure in the text (a χωρίον τετράγωνον τὸ ΑΒΓΔ) which illustrates the progressive divisions. The square ΑΒΓΔ is successively applied to the quantity 660′. Hence παραβάλλω παρά seemed to Theon the appropriate word.

Heron Metrica 1.5.25-27:
<...γίγνεται τξε· ἀπὸ τούτων ἄφελε τὰ σκε·> γίγνεται λοιπὰ ρμ· τούτων τὸ ἥμισυ· γίγνεται ο· παράβαλε παρὰ τὸν ιδ· γίγνεται ε· καὶ τὰ ιγ ἐφ' ἑαυτά· γίγνεται ρξθ.
"<...result 365. From this subtract the 225;> 140 remain as the result. Take one-half of this; result 70. Divide by 13; result 5. Multiply 13 by itself; result 169."
This problem too refers to a geometrical figure: a scalene triangle whose sides are given and whose area is to be determined. A similar usage occurs 14 times in Heron's Metrica alone.

τέμνομαι "to divide, cut" in a sense practically identical to that of διαιρέω quoted under Theon of Smyrna De utilitate 18.15-18 above. It is not used for the arithmetical operation (Mugler 413-416).

Nicomachus Intro. Arith. I 7:
περισσὸς. . .εἰς ἄνισα δύο τεμνόμενος.
"An odd number is divided into two unequal parts."

Euclid Elements 11.24.7-9:
Ἐπεὶ γὰρ δύο ἐπίπεδα παράλληλα τὰ ΒΗ, ΓΕ ὑπὸ ἐπιπέδου τοῦ ΑΓ τέμνεται, αἱ κοιναὶ αὐτῶν τομαὶ παράλληλοί εἰσιν. παράλληλος ἄρα ἐστὶν ἡ ΑΒ τῇ ΔΓ.

"Since the two parallel planes BH, ΓE are cut by the plane AΓ, the common sections are parallel. Therefore AB is parallel to ΔΓ."

FOOTNOTES

[1] This paper supplements Mugler's dictionary of Greek geometrical terminology: Charles Mugler, Dictionnaire Historique de la Terminologie Géométrique des Grecs (Paris 1958).

[2] The main exception is the sui generis Diophantus, whose relatively modern notation is described in Heath.

[3] An example might be useful. Following is a passage from Ptolemy, Almagest VI 7 (H510) as translated by G.J. Toomer (the equations refer to line segments in a diagram in the text; bracketed passages are additions by the translator):
"So let us take an eclipse for which the entry [in the table] is '15 lunar digits', i.e. one in which D, the moon's centre [at mid-eclipse] lies 1 1/4 lunar diameters inside the boundary set by the limits of the eclipse. That is to say, when

(AB-AD) = (AZ-AD) = 1 1/4 lunar diameters
and (AG-AD) = (AE-AD) = 1/4 lunar diameters.
Then, for the moon's greatest distance,
as before, AB = 56;24 minutes and AB^2 = 3180;58.
And AG = 25;4 minutes, since the moon's diameter at greatest distance is 31;20 minutes.
Therefore AG^2 = 628;20,
and, by a similar argument,
AD = [56;24-(31;20+7;50)=]17;14 minutes and
AD^2 = 296;59."

Here is an extremely literal rendering of the same passage,, beginning with "That is to say...":
"That is to say, when the [line] AD is less than either of the [lines] AB and AZ by the aforementioned one lunar diameter plus the quarter part of it, and [less than] either of the [lines] AG and AE by the quarter part of the lunar diameter; then at the moon's greatest distance, the [line] AB is equal to the aforementioned 56 24 sixtieths and the [square] of it, 3180 58. The [line] AG is 25 4 of them, since the diameter of

the moon at greatest distance is 31 20 sixtieths. And the [square] of it is 628 20. Likewise, the [line] AD is 17 14 and the [square] of it 296 59."
To some extent the difference is one of typography, but even that difference indicates the modern world's preference for schematic clarity, symbols, and well-known methods of operation. Note that there are no equal signs, no plus or minus signs, no "decimal" points, in Ptolemy's text.

[4] The following works are cited here:

Diophantus, Arithmetica
Euclid, Elements
Eudoxus, Fragments
Eutochius, Commentary on Archimedes
Heron, various works cited from the TLG CD-ROM
Pappus, Collection
Plato, Republic, Laws
Plutarch, Moralia
Theon of Alexandria, Comm. on the Almagest, Short Comm. on the Handy Tables
Theon of Smyrna, de Utilitate
Vettius Valens, Anthologiae

plus various papyri. For details of these, see the bibliography.

{note} Brackets surround an explanatory addition by the translator. One digression to elucidate this passage: 288 is one of three possibilities for the length in days of the gestation period (approx. 9 months). The astrologer is counting back 278 days from the date and hour of birth, finding the moon's position then, and postulating that degree of the zodiac as the degree in the Ascendant (i.e. just rising on the eastern horizon) when the child was born. The Ascendant governs most predictions, and it is reasonable to say that most, if not all, of the Ascendants reported in ancient horoscopes were calculated in some such way as this, not observed. Valens' ingenuity in his methods here is typical. I

will not bother to explicate all the methods found in these samples, as such explication would lead this paper too far afield.

{ W. Brashear, "Holz- und Wachstafeln der Sammlung Kiseleff (Tafel 1-13)" Enchoria 13(1985) 13-23. The horizontal separation lines are on the original tablet.

{} Each emperor had an "addition-factor" (ἡ πρόσθεσις) of either 19 or 30, given in a table in Anthologiae Book 1.17.

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