

The following is a sample calculation for Magnesium [MS-Excel 2013 is used]:

(1) Use your experimental data:

In MS-Excel, add your data and used the following formula to calculate:  $n$ ,  $\Sigma X$ ,  $\Sigma X^2$  and  $D$ .

A	B	C	D	E	F	G
2	X = [Mg] ppm	Y = Abs.	X <sup>2</sup>			
3	0.214	0.12	0.045796			
4	0.428	0.232	0.183184			
5	0.642	0.352	0.412164			
6	0.856	0.464	0.732736			
7	0.963	0.525	0.927369			
8						
9						
10	<b>Function:</b>	<b>Symbol</b>	<b>Value</b>			
11	COUNT(B3:B7)	n	5			
12	SUM(B3:B7)	SUM(X)	3.103	$\Sigma X$		
13	SUM(D3:D7)	SUM(X*X)	2.301249	$\Sigma X^2$		
14	(D11)(D13)-(D12)^2	D	1.877636			

Use: **Tool>>Data Analysis>> Regression.**

Choose: **Input Y range, Input X range, Output range location for the output data.**

SUMMARY OUTPUT						
<i>Regression Statistics</i>						
Multiple R	0.999926693					
R Square	<b>0.999853391</b>					
Adjusted R Square	0.999804521					
Standard Error	<b>0.002316431</b>					
Observations	5					
<i>ANOVA</i>						
		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression		1	0.109783102	0.109783	20459.58	7.53E-07
Residual		3	1.60976E-05	5.37E-06		
Total		4	0.1097992			
		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	
Intercept		<b>0.00304878</b>	<b>0.002564458</b>	1.18886	0.320023	
X Variable 1 (Slope)		<b>0.5406884</b>	<b>0.00378006</b>	143.037	7.53E-07	

(2) Organize your statistic data:

Based on the above output "Regression Statistics", you have the following data:

Slope:	Coefficients	<b>0.5406884</b>
Intercept:	Coefficients	<b>0.00304878</b>
S <sub>m</sub> :	Standard Error for Coefficients	0.00378006
S <sub>b</sub>	Standard Error for Coefficients	0.002564458
R <sup>2</sup>		0.999853391
S <sub>y</sub>	Standard Error	0.002316431

(3) Prepare your graph:

Choose: XY-Scatter.

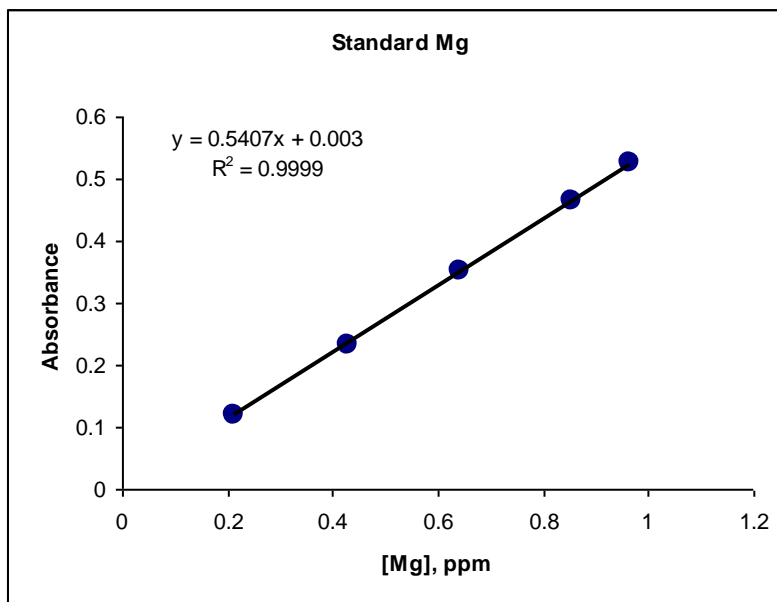
Choose: Sub-Type (Compares pairs of values). Data points without line!

On your graph click on the data-point (marker), choose "Add trend line".

Choose: Linear, (go to options), add check marks for:

Display equation on chart.

Display R-Squared value on chart.



(4) Calculate the concentration of diluted unknown, based on the Mg equation:

$$\text{Abs} = 0.5406884 [\text{Mg}] + 0.00304878$$

Using Equation of the Line to find [Mg] in the Unknown Solution	
Absorbance	[Mg] ppm, calculated
0.198	0.361
<b>0.253</b>	<b>0.462</b>
0.292	0.534

(5) Calculate uncertainty in diluted unknown concentration:

$$\sigma_x = \frac{\sigma_y}{|m|} \sqrt{\left[ \frac{1}{k} + \frac{1}{n} + \frac{(y - \bar{y})^2}{m^2 \sum (x_i - \bar{x})^2} \right]}$$

$\sigma_y$  = Standard Error

m = Slope = 0.5407

k = Number of replicate measurements

n = Number of data point = 5

$y_i$  = Absorbance of standard solutions ( $i = 1, 2, 3, \dots, 5$ )

x	y	y cal = (slope) X + (intercept)	$d_i = y - y_{cal}$	$(d_i - \bar{d})^2$
0.214	0.120	0.11876	$d_i = 0.120 - 0.11876 = 1.24 \text{ E-}3$	9.025E-7
0.428	0.232	0.2352	-3.20E-3	1.22E-5
0.642	0.352	0.35017	1.83E-3	2.372E-6
0.856	0.464	0.46661	-2.61E-3	8.41E-6
0.963	0.525	0.52373	1.28E-3	9.80E-7
<b>[Mg] STD</b>	<b>Recorded Abs.</b>	<b>Calculated Abs.</b>	<b>Average (<math>d_i</math>): <math>\bar{d} = -2.92 \text{ E-}4</math></b>	<b>Total: 2.486E-5</b>

$$\sigma_y \approx S_y = \sqrt{\frac{1}{n-2} \sum (d_i - \bar{d})^2} = \sqrt{\frac{2.486 \times 10^{-5}}{3}} = 2.878 \times 10^{-3}$$

$$\bar{y} = \frac{0.120 + 0.232 + 0.352 + 0.464 + 0.525}{5} = 0.3386$$

$$\bar{x} = \frac{\sum x}{n} = \frac{3.103}{5} = 0.6206$$

$$\sum (x - \bar{x})^2 = [(0.214 - 0.6206)^2 + (0.428 - 0.6206)^2 + (0.642 - 0.6206)^2 + (0.856 - 0.6206)^2 + (0.963 - 0.6206)^2]$$

$$\sum (x - \bar{x})^2 = [(0.16532) + (0.03709) + (4.5796 \times 10^{-4}) + (0.055413) + (0.117238)] = 0.37552$$

$$\sigma_x = \frac{0.0023164}{0.540688} \sqrt{\left[ \frac{1}{1} + \frac{1}{5} + \frac{(y - \bar{y})^2}{m^2 \sum (x_i - \bar{x})^2} \right]}$$

$$\sigma_x = \frac{0.002316}{0.5407} \sqrt{\left[1 + (0.462)^2 \left(\frac{5}{1.8776}\right) + \frac{2.3012}{1.8776} - 2(0.462) \frac{3.103}{1.8776}\right]}$$

$$\sigma_x = 4.28 \times 10^{-3} \sqrt{[1 + 0.5684 + 1.2256 - 1.5270]}$$

$$\sigma_x = 4.28 \times 10^{-3} \times 1.1256 = 4.8175 \times 10^{-3}$$

- (6) Use dilution factor ( 1:100 ) for the Mg-unknown solution the find the Original Solution Concentration:

$$\text{Therefore, for the trial 2: } 0.462 \text{ ppm} \times \frac{100}{1} = 46.2 \text{ ppm}$$

- (7) Uncertainty in original concentration:

$$\frac{\sigma_x(\text{org})}{\text{Conc. org}} = \frac{\sigma_x(\text{dil})}{\text{Conc. dil}}$$

$$\sigma_x = (\text{Conc. Org}) \times \frac{\sigma_x(\text{dil})}{\text{Conc. dil}}$$

$$\sigma_x = (46.2 \text{ ppm}) \times \frac{4.8175 \times 10^{-3}}{0.462 \text{ ppm}} = 0.482$$

- (8) Original Concentration of Mg ( $\sigma_x = 0.5$ ):

$$C_{[\text{Mg}]} \pm \sigma_x \text{ ppm}$$

$$46.2 \pm 0.5 \text{ ppm}$$

**Do same calculations in order for the Calcium and tap water!**

Calculate total hardness, using your results of the AAS measurements:

$$[\text{CaCO}_3]_{\text{ppm}} = \frac{100.1 \text{ g/mol CaCO}_3}{40.1 \text{ g/mol Ca}} \cdot [\text{Ca}^{2+}]_{\text{ppm}}^{\text{AAS}} + \frac{100.1 \text{ g/mol CaCO}_3}{24.1 \text{ g/mol Mg}} \cdot [\text{Mg}^{2+}]_{\text{ppm}}^{\text{AAS}}$$

$$[\text{CaCO}_3]_{\text{ppm}} = 2.50 \cdot [\text{Ca}^{2+}]_{\text{ppm}}^{\text{AAS}} + 4.12 \cdot [\text{Mg}^{2+}]_{\text{ppm}}^{\text{AAS}}$$