Notes on the Golden Rule capital stock in the Solow Growth Model

While a higher capital stock implies higher output, this does not mean a higher capital stock is desirable. To sustain a high capital stock, a lot of output will have to be devoted to investment, leaving less available for consumption.

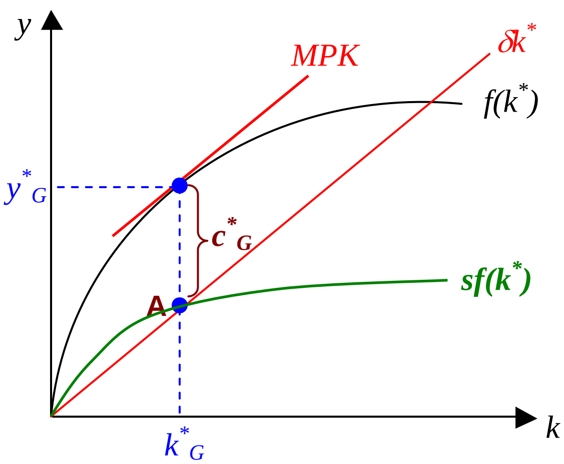
Golden Rule: The capital stock per-worker that maximizes consumption per-worker. In the Solow Growth Model with no population growth and technological progress, this occurs where

$$\boxed{MPK = \delta}$$

Recall, at steady state, investment is equal to total depreciation because savings is equal to investment.

Steady state: $sf(k^*) = \delta k^*$ Total savings = total investment: $sf(k^*) = i^*$ $\Rightarrow i^* = \delta k^*$

On the diagram below, c^* is the difference between what is produced, $f(k^*)$ and output used for investment, δk^*. On the graph, the golden rule capital stock is the k that maximizes the distance between the production function and total depreciation. Why? The difference between the two lines is consumption; the golden rule capital stock is the k that maximizes consumption. Mathematically, this is where the slope of the production function (*MPK*) is equal to the slope of the depreciation line (δ).



A country that is saving too much, has a steady state capital stock that is above k^*_G, a country that is saving too little has a steady state capital stock that is below k^*_G. Notice that only one savings rate s will ensure that the economy achieves the golden rule capital stock at steady state. On the graph, this savings rate will ensure the savings function crosses at point A on the graph.

The golden rule can be interpreted in terms of marginal product of capital and depreciation. A one-unit increase in k raises output by *MPK;* this is the added benefit of increasing k. It also implies that an extra δ units of output must be set aside to maintain the capital-labor ratio at its new higher level; this is the additional cost of increasing k. The level that maximizes consumption will be where the added benefit (*MPK*) equals the additional cost (δ).

- If $MPK > \delta$, then increase in k will increase output by *more than* the implied increase in depreciation, so consumption rises because there is more output leftover for consumption. The extra benefit (*MPK*) associated with an increase in k outweighs the cost (δ), so we should <u>increase k</u>.
- If $MPK < \delta$, then increase in k will increase output by *less than* the implied increase in depreciation, so consumption *falls* because there is less output leftover for consumption. The extra cost (δ) associated with an increase in k outweighs the benefit (*MPK*), so we should <u>decrease k</u>.

In the model with population growth and technological progress, the modified golden rule is:

$$\boxed{MPK = (n + g + \delta)}$$

The interpretation in terms of marginal product is somewhat more difficult, but the general idea is the same as in the simple model. Notice that when there is no population growth ($n = 0$) or technological progress ($g = 0$), this modified golden rule collapses to the simple one at the top of the handout.