Production Problem
Assume the production function is given by \( Y = AK^{0.4}L^{0.6} \), where \( Y \) is GDP, \( K \) is capital stock, and \( L \) is labor. The parameter \( A \) is equal to 20. Assume also that capital stock is 610 and the labor force is 2048. Capital and labor are traded in perfectly competitive markets and have the following marginal products:

\[
MPK = 8K^{-0.6}L^{0.6} \quad MPL = 12K^{0.4}L^{-0.4}
\]

a) Show that the production function has constant returns to scale. Compute total output in this economy.

b) Compute the real wage and the real rental rate.

c) Compute the share of output paid to the workers. Compute the share of output paid to the owners of capital.

d) Suppose that a technological innovation leads to an increase in \( A \), such that \( A \) increases to 25. What happens to the real wage? The real rental rate? What happens to the share of output paid to workers relative to that paid to capital owners?
Demand & Supply Problem
Assume the production function is given by \( Y = AK^{0.5}L^{0.5} \), where \( Y \) is GDP, \( K \) is capital stock, and \( L \) is labor. The parameter \( A \) is equal to 30. Assume also that capital stock is 100 and the labor force is 400. Capital and labor are traded in perfectly competitive markets. Consumption (\( C \)) is given by \( C = 600 + 0.6(Y - T) \). Investment is given by \( I = 2,000 - 10,000r \), where \( r \) is the real interest rate in percent. Taxes (\( T \)) and government spending (\( G \)) each equal 500.

a) Compute total production in this economy.

b) Using your answer to a), compute the equilibrium real interest rate, \( r \).

c) Compute the equilibrium values of consumption and investment.

d) Compute public, private, and national saving in this economy (\( S_g, S_p, \) and \( S \)).

e) Suppose government spending increases by 1,000. Compute the new values of \( C, I, S_g, S_p, \) and \( S \).

f) Using the space to the right, illustrate the effects of this increase in government spending on a savings-investment diagram. Label the axes, curves, initial equilibrium, and new equilibrium.

Additional practice problem: Suppose the government introduces a tax credit program for businesses that increases the demand for investment by 500 at any given real interest rate. Write out the new investment function, and then repeat steps e) and f) above.