Chapter 4

Real Business Cycle Theory

This section of the textbook focuses on explaining the behavior of the business cycle. The terms business cycle, short-run macroeconomics, and economic fluctuations (preferred) are used synonymously.

In general, we will study four broad classes of models to explain economic fluctuations:

- Real business cycle (RBC) theory
- Traditional/Keynesian theory
- New classical theory
- New Keynesian theory

It is essential to separate out economic models from empirical observations. For example, the Phillips curve is an empirical observation that is either explicitly incorporated into or implied by economic models. In some cases, the models will make similar predictions about the behavior of output, prices, interest rates, wages, etc. These models are distinguished based on their identifying assumptions. Therefore, a model may be rejected on the grounds of (i) its identifying assumptions, or (ii) it fails to replicate key features of the data.

Chapter 4 begins with a summary of empirical observations of the business cycle and then develops a simple RBC model.

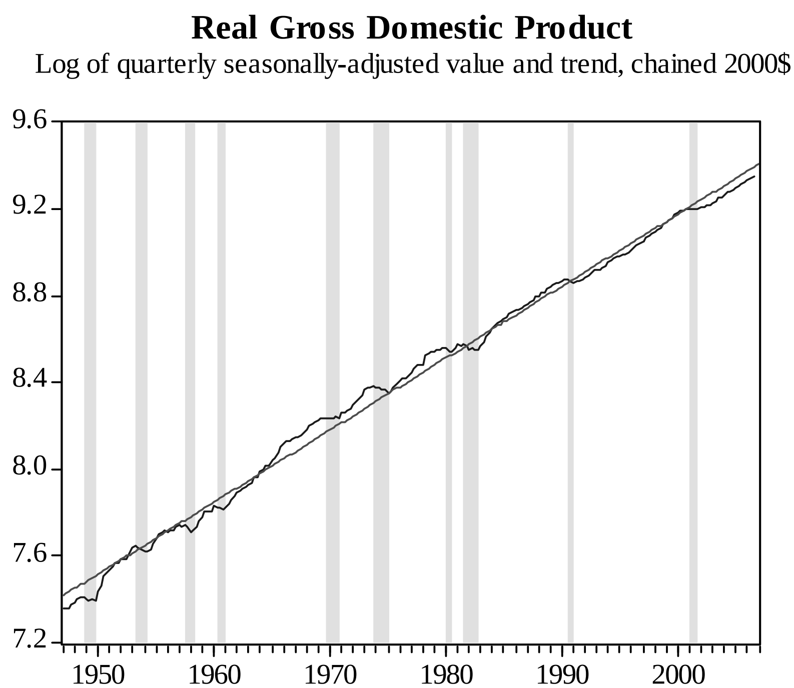
4.1 Some Facts about Economic Fluctuations

Before we build models designed to explain economic fluctuations, we need to document the key empirical facts that our models should be able to replicate.

4.1.1 Fluctuations do not exhibit regular or cyclical patterns.

Table 4.1 shows the duration and severity in business cycles. From this table, we can see that recessions vary in length (ranging between 1 and 5 quarters) and in percentage drop in real GDP (0.4% to 3.7%).

The figure below (a modified version of Figure 4.1 from Romer) shows the log of real GDP over time. We can see from this time series plot that the length between recessions varies. For example, some recessions are ten years apart, while others are only four years apart. This figure highlights the need to decompose output into two components: trend and cyclical.



Modelling implications:

- Empirical modeling of business cycles has moved away from trying to modeling recessions as occurring according to some regular time interval.

- Exogenous shocks to output may occur randomly over time (e.g., we don't need to assume they occur according to some regular, time dependent pattern).

4.1.2 Fluctuations are spread unevenly across the components of output.

The different components of expenditure account for different shares of output, but they vary greatly in their ability to explain real GDP losses during recessions. In other words, even though consumption accounts for two-thirds of expenditures, it explains only 35% of output losses during recessions. Investment explains the largest share of output losses.

The middle column of Table 4.2 reports the share of real GDP accounted for by different components of expenditures. The last column computes the percentage of the fall in real GDP associated with a specific component of expenditures. From this table we see that the following variables are most important for explaining recessions: Inventories (42%), fixed nonresidential investment (20%), and consumption of durables (15%).

This is not surprising as when people earn lower income or fear losing their jobs, they cut back on big purchases (durables), but their consumption of nondurables and services adjust less. Business cutback on production during recessions, allowing their inventories to decrease and delay purchasing new capital equipment (fixed nonresidential investment). The negative sign on net exports is not surprising. During recessions, households and businesses fewer goods, those produced at home and abroad. This translates into a reduction in imports, improving net exports. Government purchases do not change much over the business cycle. Recall, these purchases do not include transfer payments (unemployment compensation, Social Security, welfare, etc.), they only include the government's purchase of new goods and services.

Modelling implications:

- In explaining the behavior of U.S. business cycles, investment, especially inventory investment, explains most of the volatility in output. Whatever the mechanism in our economic model, investment (and to a lesser extent consumption of durables) plays a key role.
- Net exports and government purchases do not appear to be important factors in explaining U.S. business cycles. With consumption accounting for the largest share of GDP and investment explaining the largest

share of the volatility in GDP, these two components will be the focus of most demand-side models.

4.1.3 Output growth is distributed symmetrically around its mean, but there are long periods where output is above/below its trend.

We can see this from the figure above (or Figure 4.1 in Romer). Consistent with most of the economist growth models from Chapters 1-3, output appears to return to some long-run balanced growth path. In the figure above, real GDP is fitted to a log-linear trend, we would see a similar pattern for more structural measures of potential GDP (such as the Congressional Budget Office's potential GDP series), or alternative trend specifications (quadratic trend, segmented trend, etc.)

However, output does not fluctuate randomly around this trend. Specifically, periods of low (or negative) growth are often followed by periods of very high growth, while the reverse is not true. It is clear from the figure that a log-linear deterministic trend is not adequate for the purposes of identifying the cylical component of real GDP. For example, if you look at the period 1968-1980, output is above its trend value, despite NBER recessions in the early and mid 1970s.

Modelling implications:

- We can separate the trend/growth component of output from the cyclical component, allowing us to avoid integrating economic growth into our models of economic fluctuations. We abstracted from "short run" fluctuations in modeling economic growth (trend output), here we will ignore the "long run" behavior of output in modeling economic fluctuations (cyclical output).
- The asymmetry in output fluctuations suggests we may need to use more advanced econometric techniques to separate the trend/cyclical components of real GDP. A deterministic trend implies symmetry in output fluctuations that may fail to fully capture economic fluctuations.

4.1.4 Output appears to be more volatile pre-World War II.

There are two ways to see this empirical feature of output. First, if you look at the NBER recession dates, it is clear that recessions were more frequent and lasted longer during the pre-WWII era. The table below reports the duration of recessions for the 1854-2001 period and for selected sub-samples.

Sample	*From Peak to Trough (recession)*	*From Trough to Peak (expansion)*
1854 - 2001	17 months	38 months
1854 - 1919	22 months	27 months
1919 - 1945	18 months	35 months
1945 - 2001	10 months	57 months

From this table, we can see that between 1854-1919, on average, a recession occurred once every 2.25 years. By comparison, in the post-WWII era, recessions occurred only every 4.75 years (on average). One could argue that the interwar period is an anomaly (with frequent, but relatively short recessions), but even prior to WWI, recessions were more frequent than they were after 1945. We see a similar pattern with the duration of the recessions: recessions after WWII lasted an average of 10 months, compared with 22 months before 1919.

Another approach is to look at output data. The drawback of using the NBER recession dates is that we have not addressed the severity of business cycles. Unfortunately, reliable output data prior to 1929 is difficult to obtain and disaggregated output data are available only after 1947. Generally, most evidence points toward recessions being associated with larger declines in real output. Romer (1989) claims that traditional measures of real GDP are biased because they give disproportionate weight to specific sectors of the economy that tend to be more volatile. Siegler (1998) has countered this evidence, constructing a real GDP series that is consistent with the claim that real GDP is less volatile in the post-WWII period.

The Great Depression (1929-1933) is an exceptional economic event. While the U.S. experienced a longer recession and more severe output decline relative to other countries, the recession was a global economic event. While data availability is limited, we observe the recessions in the 1920s up through World War II. In the U.S., real GDP declined by 27 percent and unemployment peaked at 25 percent in 1933.

Modelling implications:

- We know that recessions are less frequent and shorter after WWII.

Also, if we accept that output is less volatile in the post-WWII era, then something must have changed. Many macroeconomists point to the use of discretionary fiscal and monetary policy.

  – With the exception of some programs introduced during the Great Depression, the federal government did not begin using fiscal policy to respond to business cycles until after WWII. The evidence on the Federal Reserve is somewhat mixed.
  – While monetary policy was subordinated to the government's needs in the 1940s, the Federal Reserve did use policy to respond to economic fluctuations in the late 1920s up until WWII.

- Based on the information above, modeling economic fluctuations may involve fundamentally different assumptions pre and post-WWII. While this may not be necessary in building models, making this assumption would not be at odds with the data. In fact, the other empirical facts documented in Chapter 4 of Romer rely on data beginning in 1947.

- Given the severity of the Great Depression, its causes are likely a confluence of factors. It remains among the most studied episodes in macroeconomics research.

4.1.5 Macroeconomic aggregates tend to fluctuate with the business cycle.

Table 4.3 in Romer reports the behavior of some key macroeconomic aggregates during recessions. These variables are chosen because they may (or may not, depending on the model) affect the behavior of output in our economic models of the business cycle. From the table, we see the following:

- Procylical variables (strong positive correlation with cyclical output): employment, average weekly work hours, output per hour, nominal interest rates,

- Countercylical variable (strong negative correlation with cyclical output): unemployment rate

- Weak/no correlation with output: real compensation (wages), inflation ex post real interest rates, real money stock

During recessions, total employment (number of people working), average work hours, and productivity tend to decline. While the link between productivity is somewhat weaker than employment and average work hours, there is clearly a positive relationship between GDP and productivity. This is consistent with the negative correlation between unemployment and GDP. This strong link between unemployment and output growth is summarized by Okun's Law:

$$\%\Delta GDP = k - c\Delta U$$

where k is the long-run growth rate of real GDP (usually assumed to be about 3%), U is the unemployment rate, and c measures the correlation between unemployment and the growth rate of real GDP (about 2%):

$$\%\Delta GDP = 3 - 2\Delta U$$

The relationship between interest rates/money and GDP is somewhat weaker than these labor market variables - interest rates seem to be more closely linked to economic fluctuations than the money supply. Nominal interest rates seem to be more closely linked to recessions than do real interest rates. Real wages, real interest rates, and real money stock are only weakly correlated with output. Inflation appears to be uncorrelated with recessions.

Some of the variables in Table 4.3 of Romer are lagging (unemployment), coincident (employment), or leading (money supply) economic indicators of the business cycle.[1]

Modelling implications:

- It is important to consider the labor-market implications of our economic models. While models of economic fluctuations are focused on explaining the behavior of output, the aggregate labor market is derived from the economy-wide market for final goods and services.
- The lack of a relationship between wages and GDP causes problems in modeling economic fluctuations and points to a general lack of understanding of the aggregate labor market.
  - Most of the models we will study imply a stong correlation between output and real wages (either positive or negative). Developing models that break this strong link is difficult because shocks are assumed to either affect labor supply or labor demand (and hence equilibrium wages).

[1] The Conference Board publishes indices of economic indicators. While the data are proprietary, one could reconstruct the indices according to the weights assigned by the Conference Board.

  - While there is a weak negative relationship, models that rely on a strong negative link between real wages and output may be suspect.

- The empirical evidence on the money supply indicates that while changes in the money supply may be important, they are likely not the only cause of recessions.

  - These observations are consistent with possible monetary policy responses to recession. The Federal Reserve could "lean against the wind" (attempt to offset the effects of a recession by increasing money supply) or,
  - the Federal Reserve may opt to do nothing in response to recession, in which case it is likely that the money supply would decline as money demand declines with a drop in income.

4.2 Theories of Fluctuations

It is natural to think of market failures/frictions in modeling economic fluctuations, but this isn't absolutely necessary. A Walrasian model is one with competitive markets and no market imperfections (e.g., no asymmetric information, externalities, etc.) Beginning with a neoclassical model (e.g., a Solow/Diamond-type model without growth), we can relax the following assumptions:

1. Incorporating a source of shocks
   We need some mechanism for output to deviate from its balanced growth path (this is consistent with 4.1.1 and 4.1.3 above). Since technology is exogenous in the neoclassical growth model, this is a natural place to start. Government purchases is another possible sources of shocks in the neoclassical growth model.

2. Allow for variations in employment
   In the neoclassical growth model, labor supply is exogenous. So, while labor demand could fluctuate with technology shocks (see #1 above), labor supply is assumed to be fixed at a given point in time (or growing at a constant rate over time). One natural way to incorporate variations in labor supply is to make the decision to work part of the household's utility maximization problem.

This is the approach adopted by real business cycle (RBC) theory: to keep the fundamental assumption of a Walrasian model where households maximize utility, perfectly-competitive firms maximize profits, and all agents access to the same information.[2]

Traditional Keynesian models (such as the IS/MP model studied in Chapter 5) essentially scratch the underlying assumptions of the neoclassical growth model in favor a non-Walrasian approach to modeling business cycles. The primary criticism of the traditional Keynesian model is that it does not provide a theory for why variables behave the way that they do. For example, one key feature of the traditional Keynesian model is that households save a constant fraction of their income - but why? Why don't households consider future earnings in their consumption decision? Another key feature is that wages are fixed. Why? There is no mechanism for answering these questions in a traditional Keynesian model.

The new Keynesian models studied in Chapter 6 attempt to address these questions by developing models based on microfoundations of household and firm behavior. The most common way to explain sticky prices/wages is to assume imperfect competition in the production of final goods coupled with some costs associated with changing prices.

Another approach that we will study in Chapter 6, new Classical theory, assumes asymmetric information, but otherwise retains the assumptions of the neoclassical model. In this approach, incomplete price adjustment is explained by a lack of information by the firms.

While many macroeconomists disagree with the basic premise that economic fluctuations are caused by technology shocks, the contributions of real business cycle theory are significant. RBC theory prompted a movement toward modeling macroeconomic behavior based on microfoundations of household and firm behavior. Models that do not incorporate this, regardless of whether they are RBC or not, are problematic because they do not fully explain the endogenous decisions of households and firms.

This is related to the Lucas (1976) econometric critique, one of the most important research papers in modern economics. The critique has far reaching implications for empirical work and model building in macroeconomics. Lucas (1976) claims that we cannot use a system of equations to conduct policy experiments because the underlying behavior of households and firms will change in response to the policy itself. For example, consider the tra-

[2] RBC models allow for stochastic shocks to affect output, employment, etc. The critical assumption is not that information is known with certainty, but that all agents have access to the same information.

ditional Keynesian assumption that households save a constant fraction of their income (without some underlying model of how/why they do so). Now, suppose the Federal Reserve announces an increase in money supply. Surely households will increase their consumption today, realizing that future income will increase as a result of this policy.

In terms of empirical work, Lucas (1976) has generated a need for distinguishing between anticipated versus unanticipated shocks to the economy. If the shock is unanticipated, then households will not have time to adjust their behavior. How do we identify anticipated versus unanticipated shocks? The most common way is to use vector autoregressions (VARs) that you will study in ECON 200C.

4.3 A Baseline Real-Business Cycle Model

Assumptions

- Discrete-time model where households and firms live forever.
- Perfectly-competitive firms choose capital K and labor L to maximize profits.[3]
- A fraction of the exisiting capital stock, δ, depreciates each period.
- Households own the factors of production.
- Household choose consumption and labor to maximize lifetime utility.
- Households are endowed with one unit of time to be divided between labor ℓ and leisure $(1-\ell)$.
- Technology (cyclical component only) evolves over time according to an $AR(1)$ process.

Production

The firm's production function is:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

[3] We can relax the assumption of a Cobb-Douglas production function. We only need to assume constant returns to scale for the general implications of the model to hold.

where $0 < \alpha < 1$. The law of motion for the capital stock is:

$$K_{t+1} = K_t + I_t - \delta K_t$$

We will normalize the price of output to be equal to one. Let w_t and r_t^d denote the real wage and real rental rate (excluding depreciation) at time t :

$$\text{Profit} = \Pi = K_t^{\alpha}(A_t L_t)^{1-\alpha} - w_t L_t - r_t^d K_t$$

The FOCs for this problem are:

$$\begin{aligned} \frac{\partial \Pi}{\partial K_t} &= 0 : \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} - r_t^d = 0 \\ \frac{\partial \Pi}{\partial L_t} &= 0 : (1-\alpha)\, K_t^{\alpha} {A_t}^{1-\alpha} L_t^{-\alpha} - w_t = 0 \end{aligned}$$

The firm knows it will lose δK_t each period when it choses the capital stock. For each unit of capital the firm purchases, it will lose δ units next period. Let r_t denote the real rental rate including this cost. This yields the following input prices:

$$\begin{aligned} w_t &= (1-\alpha)\left(\frac{K_t}{A_t L_t}\right)^{\alpha} A_t = (1-\alpha)\,\frac{Y_t}{L_t} \\ r_t &= \alpha\left(\frac{A_t L_t}{K_t}\right)^{1-\alpha} - \delta = \alpha \frac{Y_t}{K_t} - \delta \end{aligned}$$

The expressions above can be rewritten to solve for L_t and K_t. These expressions give us the demand for capital and the demand for labor.

Technology

The cyclical component of technology, $\tilde{A}_t$, evolves according to an $AR(1)$ process:[4]

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_t$$

where $\varepsilon_t \overset{iid}{\sim} (0, \sigma_\varepsilon^2)$ and $-1 < \rho_A < 1$.

Romer incorporates government purchases into the model in much the same way, but we will ignore them here.

Labor Supply

[4] This can be found empirically by taking a Solow residual and fitting the (log of) residual itself to a trend.

The population N grows at rate n :

$$\ln(N_t) = \overline{N} + nt$$

For simplicity, we will assume that there is one household.[5] Note that if we assume the population is the working-age population and that labor force participation is 100%: $N_t = L_t = \ell_t$. Relaxing this assumption does not change the model's basic implications. We could easily assume that a fraction of N_t is actually working. We make the distinction between L_t (labor demand) and ℓ_t (labor supply) to track which is a household choice (ℓ_t) versus a firm choice (L_t). With market clearing, we know that the quantity of labor demanded equals the quantity supplied.

4.4 Household Behavior

The inifitely-lived household maxmizes expected value of lifetime utility (subject to the lifetime budget constraint):

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - \ell_t)$$

where $0 < \beta < 1$ is the discount factor[6], c_t is consumption in period t, and $1 - \ell_t$ is leisure time. Note that ℓ_t is time spent working, with ℓ_t entering negatively into the utility function above. The household generates income from working, earning w_t, and renting capital at r_t.

Imposing a log-utility function, we can express utility each period as:

$$u_t = \ln c_t + b \ln(1 - \ell_t)$$

where $b > 0$. This is done to allow for the household to place different values on consumption and leisure time.

4.4.1 Intertemporal Substituion in Labor Supply

The household maxmizes utility subject to the budget constraint.

[5] For simplicity, we will assume that there is one household. In Romer, N/H is the number of people in the households. Since H is a constant, this assumption will not change the general results.

[6] Romer uses $\beta = e^{\rho}$ in line with the log-linear structure of the model. Both ways of expressing the discount factor are common in research papers.

Case 1: Household lives for one period

The log utility function is given above. The budget constraint is simple. Since the household cannot save its income (earning interest r_t) it only earns $w_t \ell_t$ from working. Since the household does not save, the budget constraint is:

$$c_t \leq w_t \ell_t$$

Therefore, the household maximization problem can be expressed as a Lagrangian:

$$L = \ln c_t + b \ln(1 - \ell_t) + \lambda_t (w_t \ell_t - c_t)$$

where the household chooses c_t and ℓ_t to maximize utility. The FOCs for this problem are:

$$\begin{aligned}
\frac{\partial L}{\partial c_t} &= 0 : \frac{1}{c_t} - \lambda_t = 0 \\
\frac{\partial L}{\partial \ell_t} &= 0 : b \frac{1}{(1 - \ell_t)}(-1) + \lambda_t w_t = 0 \\
\frac{\partial L}{\partial \lambda_t} &= 0 : w_t \ell_t - c_t = 0
\end{aligned}$$

From the first two FOCs we have:

$$\frac{1}{c_t} w_t = b \frac{1}{(1 - \ell_t)}$$

This expression shows the household's labor-leisure choice. On the lefthand side, working one more unit of time increases utility from added consumption ($\frac{1}{c_t}$ for each unit of consumption multiplied by the total units purchased, w_t). On the righthand side, the household forefeits utility from giving up one unit of leisure time - the marginal utility of leisure is $b \frac{1}{(1-\ell_t)}$. Combining these FOCs, we arrive at the following expression:

$$\frac{1}{\ell_t} = b \frac{1}{(1 - \ell_t)}$$

Notice that the household's labor choice is independent of the wage in this case.

Case 2: Household lives for two periods

The budget constraint is now:[7]

$$c_t + \frac{c_{t+1}}{1+r_{t+1}} \leq w_t \ell_t + \frac{w_{t+1}\ell_{t+1}}{1+r_{t+1}}$$

This should look familiar to you from the Diamond mode, except that here, the household can work in period $t+1$ (period 2).Lifetime utility is:

$$U = \ln c_t + b\ln(1-\ell_t) + \ln c_{t+1} + b\ln(1-\ell_{t+1})$$

The Lagrangian is:

$$\begin{aligned} L &= \ln c_t + b\ln(1-\ell_t) + \beta\left[\ln c_{t+1} + b\ln(1-\ell_{t+1})\right] \\ &\quad +\lambda_t\left(w_t\ell_t + \frac{w_{t+1}\ell_{t+1}}{1+r_{t+1}} - c_t - \frac{c_{t+1}}{1+r_{t+1}}\right) \end{aligned}$$

where the household has four choice variables:$c_t, c_{t+1}, \ell_t, \ell_{t+1}$. The FOCs for this problem are:

$$\begin{aligned} \frac{\partial L}{\partial c_t} &= 0 : \frac{1}{c_t} - \lambda_t = 0 \\ \frac{\partial L}{\partial c_{t+1}} &= 0 : \beta\frac{1}{c_{t+1}} - \lambda_t\frac{1}{1+r_{t+1}} = 0 \\ \frac{\partial L}{\partial \ell_t} &= 0 : b\frac{1}{(1-\ell_t)}(-1) + \lambda_t w_t = 0 \\ \frac{\partial L}{\partial \ell_{t+1}} &= 0 : b\beta\frac{1}{(1-\ell_{t+1})}(-1) + \lambda_t\frac{w_{t+1}}{1+r_{t+1}} = 0 \end{aligned}$$

Combining the first two FOCs:

$$\frac{1}{c_t} = \beta(1+r_{t+1})\frac{1}{c_{t+1}}$$

This is the familiar Euler equation from the Diamond model. The third and fourth FOCs can be combined to yield:

$$\frac{(1-\ell_t)}{(1-\ell_{t+1})} = \frac{1}{\beta(1+r_{t+1})}\frac{w_{t+1}}{w_t}$$

[7] Here, I've used the notation t to denote period 1 and $t+1$ to denote period 2. When we generalize the model to allow for inifinite periods, the notation will be the same. The household will choice a consumption path $(c_t, c_{t+1,}c_{t+2}...)$ and employment path $(\ell_t, \ell_{t+1,\ell t+2}...)$ and these paths can be expressed in terms of choices made in time t and $t+1$.

This expression reveals how the household will substitute labor across time. For example, suppose that $w_t > w_{t+1}$ because of a positive technology shock today. How does the household respond? Higher wages today induces households to work more today, so households substitute toward working when their wages are highest: $\ell_t > \ell_{t+1}$. Second, there is an income effect.

Now consider an increase in the real interest rate. In this case, the household will respond by working more today. This will allow her to save more, and generate more interest on savings.

It is important to note in the expression above that households respond to changes in their relative wages and the interest rate (changes in r_{t+1} effectively change the relative benefit of working today versus tomorrow). This mechanism is known as the intertemporal substitution in labor supply. In the Diamond model, we effectively forced the household to work only in period 1. Here, we allow her to shift working across the two periods through the assumption that leisure time improves utility.

Finally, notice that if we allowed the households to life forever, the basic relationships would be the same. For the household making a choice at time t, what matters is how she substitutions between consumption today versus tomorrow, and working today versus tomorrow.

4.4.2 Household Optimization under Uncertainty

Case 3: Household lives for two periods and there is uncertainty

In this case, the household has to make her consumption/leisure choices based on expected future income. The budget constraint and utility function are the same as in case 2, but there is an expectations operator E_t on future values. The notation E_t means the household is making a choice conditional on information available at time t :

$$\begin{aligned} L &= \ln c_t + b\ln(1-\ell_t) + \beta E_t\left[\ln c_{t+1} + b\ln(1-\ell_{t+1})\right] \\ &\quad + \lambda_t\left(w_t\ell_t + E_t\left[\frac{w_{t+1}\ell_{t+1}}{1+r_{t+1}}\right] - c_t - E_t\left[\frac{c_{t+1}}{1+r_{t+1}}\right]\right) \end{aligned}$$

Working from the FOCs, we have the following Euler equation:

$$\frac{1}{c_t} = \beta E_t\left[(1+r_{t+1})\frac{1}{c_{t+1}}\right]$$

We cannot separate out the two terms on the right-hand side because c_{t+1} and r_{t+1} may be correlated. To get some intuition as to how technology shocks affect the household decision, note the following definition of the covariance between two random variables, X and Y:

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

We can use the definition of the covariance and apply it to our Euler equation:

$$E_t\left[(1+r_{t+1})\frac{1}{c_{t+1}}\right] = \beta\left\{Cov_t\left[(1+r_{t+1}), \frac{1}{c_{t+1}}\right] + E_t\left[(1+r_{t+1})\right]E_t\left[\frac{1}{c_{t+1}}\right]\right\}$$

Plugging this definition into our Euler equation:

$$\frac{1}{c_t} = Cov_t\left[(1+r_{t+1}), \frac{1}{c_{t+1}}\right] + E_t\left[(1+r_{t+1})\right]E_t\left[\frac{1}{c_{t+1}}\right]$$

Now, we have a few cases we can study. Below, we will consider how an increase in the interest rate affects the household consumption decision.

1. $Cov_t\left[(1+r_{t+1})\frac{1}{c_{t+1}}\right] = 0$
   In this case, we can rewrite the expression as:

   $$\frac{1}{c_t} = E_t\left[(1+r_{t+1})\right]E_t\left[\frac{1}{c_{t+1}}\right]$$

   This will give us a benchmark to analyze the other two cases. Here, when the interest rate increases, the righthand side exceeds the left-hand side of the expression above. To satisfy the Euler equation, the household needs to shift consumption away from the current period (increasing $\frac{1}{c_t}$) and toward the future period (reducing $\frac{1}{c_{t+1}}$). The substitution effect dominates.

2. $Cov_t\left[(1+r_{t+1})\frac{1}{c_{t+1}}\right] > 0$
   This implies return to savings is high when the marginal utility of consumption is high (e.g., when c_{t+1} is low). Here, to maintain the Euler equation, the household will need to shift even more current consumption into the future (relative to what was discussed in #1 above). The subtitution effect dominates.

3. $Cov_t\left[(1+r_{t+1})\frac{1}{c_{t+1}}\right] < 0$
   Here, the return to savings is high when the marginal utility of consumption is low (e.g., when c_{t+1} is high). Here, to maintain the Euler

equation, the household will need to shift less current consumption into the future (relative to what was discussed in #1 above). If this covariance is large enough, current consumption could potentially increase in response to an increase in the interest rate. This would be a situation where the income effects dominate the substitution effects, causing households to consume more today because of the anticipated increase in lifetime income. However, it is possible for the substitution to dominate in this case - only in cases when the covariance is very large can the income effect dominate.

We will come back to this covariance term in discussing consumption theory in Chapter 7.

The Tradeoff between Consumption and Labor Supply

From the Lagrangian above, we have the following FOC for labor supply:

$$\frac{\partial L}{\partial \ell_t} = 0 : b\frac{1}{(1-\ell_t)}(-1) + \lambda_t w_t = 0$$

Combining this with the FOC for current consumption $(\lambda_t = \frac{1}{c_t})$:

$$\begin{aligned} b\frac{1}{(1-\ell_t)} &= \frac{1}{c_t}w_t \\ \frac{c_t}{(1-\ell_t)} &= \frac{w_t}{b} \end{aligned}$$

Notice that uncertainty does not enter into the household's labor choice at time t.

4.5 Special Case of the Model

There is no closed-form solution to the model above. This section makes some simplifying assumptions so that we can study the behavior of variables over time in response to technology shocks.

4.5.1 Simplifying Assumptions

- 100% depreciation ($\delta = 1$)
- No government spending

This implies that capital each period is equal to investment:

$$K_{t+1} = Y_t - C_t$$

and that the rental rate is equal to the marginal product of capital (so that δ drops out of the firm's profit maximization problem):

$$(1 + r_t) = \alpha \left(\frac{A_t L_t}{K_t} \right)^{\alpha}$$

4.5.2 Solving the Model

There are two ways to solve the model. First, we can use the market equilibrium approach - essentially solving for the equilibrium in several different markets at the same time. Second, we can assume the outcome is Pareto optimal (as is the case with a competitive equilibrium with no market imperfections), and is therefore the result of a social planner maximizing social welfare. Romer opts to solve the model as a competitive equilibrium. This is considerably more difficult, than solving the social planner's problem. Also, the social planner approach is the one most often used in RBC research.

Solving the model mathematically goes beyond the scope of our class, we can use the FOCs from above to get most of the intuition for how macroeconomic variables behave in response to shocks.

4.6 Solving the Model in the General Case

The most common approach is the take the equilibrium conditions and use a log-linear approximation (around some steady state). This allows us to express the model's equilibrium as a system of first-order difference equations. This is important because it suggests we are able to condense the decisions of households and firms into a set of equations that are relatively simple. See Hartley, Hoover, and Salyer (1998) if you are interested in learning more about how to solve these models. While the notation differs slightly from Romer, it is a more practical guide for how to solve these models.

4.7 Implications

If we choose parameter values, we can generate simulated time series data that are consistent with the equilibrium conditions from the RBC model. The important thing to note is that technology shocks drive business cycles. These shocks are what push output, consumption, investment, work hours, etc., away from their long-run trend values. Using the simulated data, we can compare the standard deviations and correlations from the simulated data to those found in actual time series data.

Using the first-order difference equations, we can analyze how a one-time shock to technology affects macroeconomic variables.

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