Assignment #2
Due Monday, March 3rd @ 5:30pm

This second assignment covers aggregate production and economic growth. This assignment lays some important theoretical groundwork for understanding more advanced models in the economic growth literature.

1. **Transition Dynamics in the Basic Solow Growth Model**
For each of the following cases, you should illustrate the basic Solow diagram (with no technological progress) showing the initial and new steady state. Include an impulse response graph for each of the following to show how the economy returns to the balanced growth path will change: capital per worker, output per worker, the real wage, and the real rental rate.
   (a) The rate of depreciation falls.
   (b) The population growth rate rises.

2. **Technological Progress**
Consider a standard Solow growth model with a Cobb-Douglas production function with labor-augmenting technological progress. Assume that technological progress grows at rate g.
   (a) Express the production function in per worker terms. Working from this expression, derive the steady state condition for the Solow growth model with labor-augmenting technological progress. [NOTE: This requires computing time derivatives.]
   (b) Find expressions for k^*, y^*, and c^* as functions of the parameters of the model, $s,n,d,g,$ and α. What is the golden-rule value of k? Solve for this value in terms of the parameters in the model.
   (c) What saving rate is needed to yield the golden-rule capital stock? Explain.
   (d) What is the benefit of using this model versus the basic model? Cite limitations of the basic model that are resolved by the incorporation of technological progress.

3. **The Golden Rule**
Consider a Solow growth model with Cobb-Douglas production, savings rate s, depreciation rate δ, population growth rate n, and rate of technological progress equal to g. You can use your answers from Question 2 to answer the following questions.
   (a) Consider the following empirical observations for the U.S.:
   Capital stock is 2.5 times GDP — Population growth is roughly 2%
   Depreciation accounts for 10% of GDP — GDP grows at a rate of 3%
   Capital owners' share of output is roughly 30%
   Based on these data, is the U.S. currently at the golden rule level of capital?

   (b) Based on these data, what is the golden rule level of capital?
   (c) Suppose the U.S. government implements a policy that achieves the savings rate needed to achieve the golden rule level of capital. Using impulse responses, illustrate how the following variables would change as the U.S. transitions to its new balanced growth path: capital, output, and consumption.

4. Natural Resources

This problem expands the standard Cobb-Douglas production function, by incorporating another input: land, T. The aggregate production function for the economy is given by the following function, where T represents a fixed supply of land and L represents work input and with $\alpha,\gamma > 0$: $Y = AK^{\alpha}T^{\gamma}L^{1-\alpha-\gamma}$

(a) Under what conditions will this production function exhibit constant returns to scale? Show that this condition implies constant returns to scale production.
(b) Let p = price of output, w = nominal wage, r = nominal rental rate, and q = nominal rent paid on natural resources. Set up the firm's profit maximization problem.
(c) Find the first order conditions for this problem. Is the natural resource input paid its marginal product in this economy?
(d) Compute the shares of income paid to capital, labor, and land.
(e) How does the incorporation of land affect the balanced growth path for this economy? Explain.
(f) How would your answer to (e) change if we were dealing with a nonrenewable natural resource. Explain the intuition for the difference versus what you found in (e).

5. Diamond Model

Assume households have the following lifetime utility function:

$$U_t=\log(C_{1,t})+\beta\log(C_{2,t})$$

Households earn wages *w* in the first period when they are working. The households divide their wages between consumption in period 1, $C_{1,t}$ and savings S. Each unit the household saves earns interest rate *r*, paid out in the second period. In the second period, they retire, and must rely on their savings to finance consumption in this period. Note, the β is the discount factor (equivalent to (1/(1+ρ)) in D. Romer's presentation of the Diamond model).

(a) Derive the Euler equation for this economy.
(b) Solve for the steady state level of consumption in the first period and second period.
(c) Solve for the steady state savings rate. Describe how households make savings decisions in the Diamond Model and relate this to your answers to (b).
(d) Compare and contrast your findings from (c) to the assumptions we made in the standard Solow model.

6. Social Security

Using the same utility function as the one in Question 5, this question will analyze the differences between a pay-as-you-go social security system. In this system (identical to the one we have today), each member of the young generation makes a transfer payment T to the old generation who receive (1+n)T.

(a) Write down the budget constraint for the overlapping generations with this program.
(b) Assume that $r = n$, what is the effect of this program on consumption and savings? How does your answer change if $r \neq n$?
(c) How will this pay-as-you-go system affect the steady state level of capital?
(d) Now, suppose instead of a pay-as-you-go system, the government imposes a fully-funded social security program. In this program, the young generation's contribution T are invested by the government and yield a total return of (1+r)T when the young generation becomes old. Write out of the budget constraint with this program.
(e) How does the fully-funded system affect the steady state level of capital?