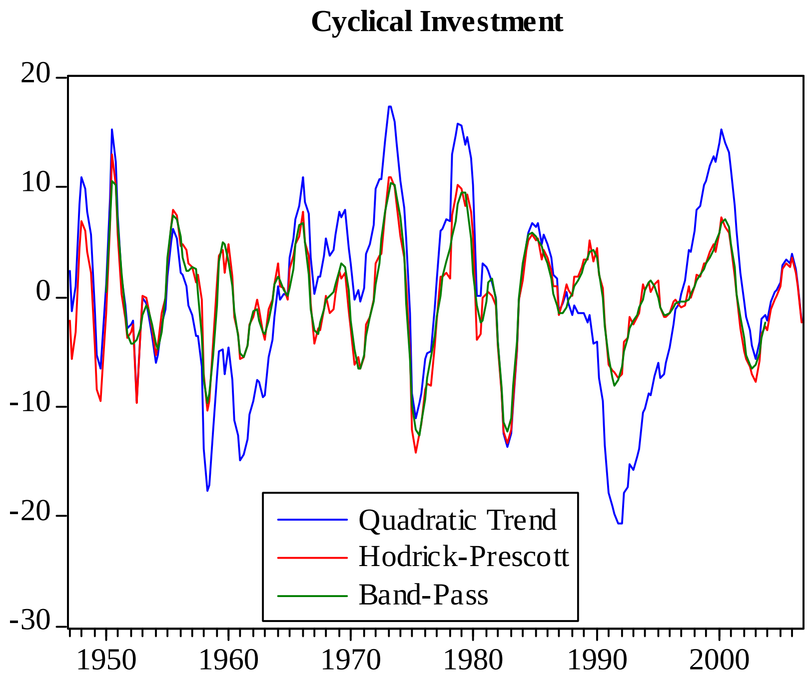
ECON 200A
Spring 2008

Answers to Assignment #4

1. **Business Cycle Data**

   (a) See the figure below.



From the figure above, we see that the quadratic trend technique generates a cyclical series that is relatively volatile. After the late 1980s, this pattern becomes very obvious. The HP and band-pass filters generate very similar cyclical series. The quadratic trend technique assumes that the log of investment follows a quadratic trend, then extracts the residual from the following regression:

$$\ln(i_t) = \alpha + \beta t + \gamma t^2 + \varepsilon_t$$

The HP filter uses an ad-hoc formula to extract the cyclical series. The band-pass filter uses the frequency of cycles to extract the cylical component of investment. The benefit of the filter approach is that it does not assume a specific linear process for a time series.

   (b) See table below. Your values for the U.S. data may differ slightly because I downloaded the data some time ago. This would not change the general conclusions below.

	U.S. Data *(1947:1-2006:4)*	U.S. Data *(from Romer)*	Baseline RBC
σ_Y	1.58	1.92	1.30
σ_C/σ_Y	0.77	0.45	0.31
σ_I/σ_Y	3.06	2.78	3.15
σ_L/σ_Y*	1.174	0.96	0.49
Corr(L,Y/L)*	0.18	-0.14	0.93

Note: Use of the band-pass filter implies the cyclical series begin two years after the sample period given above.
* Work hours data available beginning in 1964

(c) The general conclusions are the same: Consumption is less volatile than output, investment is more volatile than output, and work hours have similar volatility than output. One exception is that our sample exhibits a weak positive correlation between work hours and productivity (whereas the Romer sample finds a weak negative correlation). Our figures differ from Romer's because the sample period is different (Romer's are based on figures from prior to 1992) and because we use the band-pass filter to obtain the cyclical series - Romer's figures used the HP filter (see text for a complete explaination of how Hansen and Prescott (1992) used the filter).

(d) #1: The baseline RBC model implies that work hours are significantly less volatile than output. In the data, both in our sample and Hansen and Prescott's, we see that work hours have about the same volatility as output. The reason for relatively smooth work hours is that the baseline model uses a log utility function. It is difficult to generate a result where households reduce/increase work hours by large amounts, in response to a technology shock.
#2: The baseline RBC model generates a strong positive correlation between work hours and productivity. In our sample, we observe a weak positive correlation, and Hansen and Prescott observe a weak negative correlation. In the data, the correlation because work hours and productivity is not statistically different from zero. In the RBC model, we generate this result because technology shocks affect labor demand. Suppose there is a positive technology shock. This increases labor productivity, increasing labor demand. This increases wages, inducing households to work more. Since the shocks only affect the labor demand curve, it is very difficult to break this strong positive correlation between productivity and work hours.

2. **Standard RBC Model and Labor Supply**
See the reading guide for Chapter 4 of Romer for more details. Here, I will work directly from the first order conditions presented in the guide to avoid repetition.

(a) See reading guide.

(b) See reading guide for derivation. The Euler equation is:

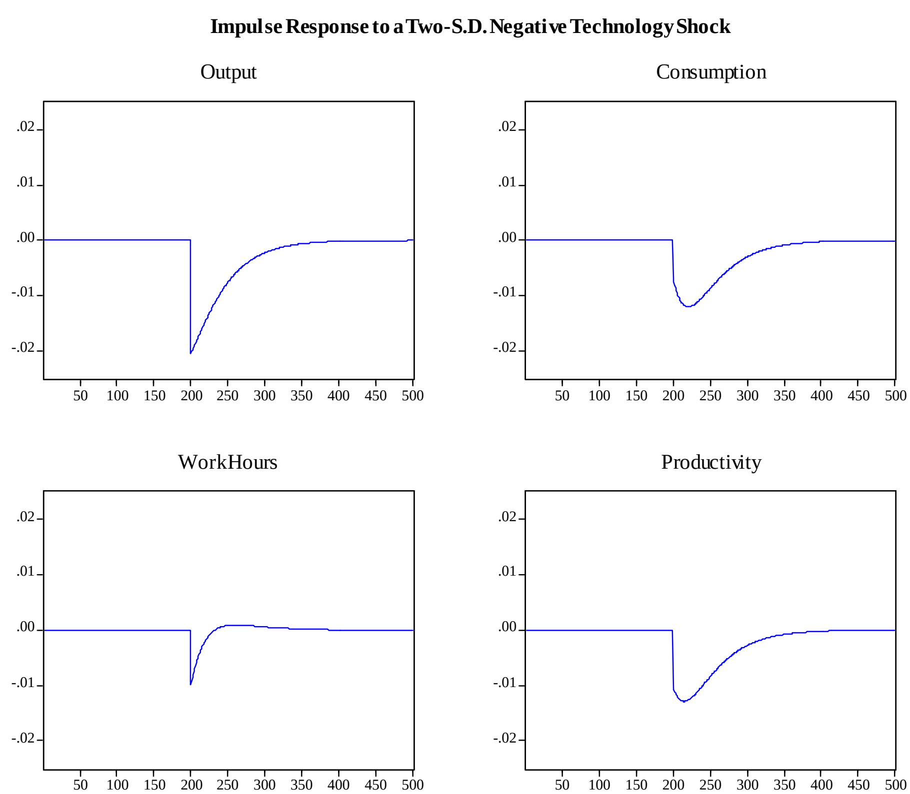
$$\frac{1}{c_t} = \beta(1+r)\frac{1}{c_{t+1}}$$

(c) See reading guide for derivation. From the FOCs (for current consumption and current work hours) we can obtain the following expression:

$$\begin{aligned} b\frac{1}{(1-\ell_t)}(-1) + \frac{1}{c_t}w_t &= 0 \\ \frac{1}{c_t}w_t &= b\frac{1}{(1-\ell_t)} \end{aligned}$$

Suppose the household decides to work one more unit of time. The term on the right hand side measures the additional utility $\left(\frac{1}{c_t} \text{ from one unit of consumption}\right)$ she receives from purchasing w_t units of consumption (since she earns w_t for working one unit of time). The term on the righthand side is the marginal utility she forgoes when she enjoys one unit less of leisure time. Notice this term is equal to the marginal utility of leisure (differentiating the utility function with respect to $(1-\ell_t)$) - we can think of this is the marginal disutility from working one more unit of time.

(d) I provided the impulse responses in case you curious what they would look like in this model. This might help to see how the model differs from the Hansen model in the next question, as well as the limitations of the RBC model addressed in above. In these examples, the shock occurs in period 200 and the impulse is limited to show the response 300 periods after this shock. For the same reasons discussed in Question 1(d) above, work hours and productivity decline during recession (which is casued by a negative technology shock). When there is a negative technology shock, firms' demand for labor falls (because labor productivity declines -MPL falls), leading to a decrease in the real wage. This causes a reduction in the quantity of work hours supplied by households. Also, we see that the decline in output is greater than the decline in work hours; consistent with the failures of the RBC model pointed out in Question 1.



3. **Hansen (1985)**
This question considers a variant on the model from the previous question. Assume the problem is the same, except now the household's lifetime utility is defined as:

$$U = \ln(c_t) + b(1-\ell_t) + \beta\left[\ln(c_t) + b(1-\ell_t)\right]$$

Here, leisure is a linear in the utility function.

(a) The Lagrangian for this problem is:

$$\begin{aligned}\mathcal{L} &= \ln(c_t) + b(1-\ell_t) + \beta\left[\ln(c_{t+1}) + b(1-\ell_{t+1})\right] \\ &\quad +\lambda_t\left(w_t\ell_t + \frac{w_{t+1}\ell_{t+1}}{1+r} - c_t - \frac{c_{t+1}}{1+r}\right)\end{aligned}$$

where the household has four choice variables:$c_t, c_{t+1}, \ell_t, \ell_{t+1}$. The FOCs for this problem are:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial c_t} &= 0 : \frac{1}{c_t} - \lambda_t = 0 \\ \frac{\partial\mathcal{L}}{\partial c_{t+1}} &= 0 : \beta\frac{1}{c_{t+1}} - \lambda_t\frac{1}{1+r} = 0 \\ \frac{\partial\mathcal{L}}{\partial \ell_t} &= 0 : b(-1) + \lambda_t w_t = 0 \\ \frac{\partial\mathcal{L}}{\partial \ell_{t+1}} &= 0 : b\beta(-1) + \lambda_t\frac{w_{t+1}}{1+r} = 0\end{aligned}$$

The Euler equation is the same as in the baseline RBC model in Question 2:

$$\frac{1}{c_t} = \beta(1+r)\frac{1}{c_{t+1}}$$

(b) Expressing current consumption as a function of current work hours:

$$\frac{1}{c_t}w_t = b$$

Therefore, the marginal utility of leisure (or marginal disutility of working) is constant. To see how this affects household work decision, suppose that $\frac{1}{c_t}w_t > b$. This would mean the marginal utilty of one more unit of consumption exceeds the marginal utility from leisure, so the household will work. If $\frac{1}{c_t}w_t < b$, then the household receives more utility from leisure than it would from added consumption (generated from working), so the household will not work. If $\frac{1}{c_t}w_t = b$ then the household is indifferent between working and not working. Notice that the household either works one unit of time, or does not work at all - this model is often referred to as an indivisible labor model. The household's labor-leisure choice is discrete, rather than continuous (as it is in the standard RBC model).

(c) Suppose that currently, the household is not working, so that $\frac{1}{c_t}w_t < b$. Then, there is a positive technology shock that increases w_t to w_t' such that $\frac{1}{c_t}w_t' > b$. Now, the household will work. If we compare this to what happens in the standard RBC model from Question 2, we see that the household decision in Question 2 is more flexible. In that model, the household will respond to the positive technology shock by increasing work hours. The indivisible labor model is thought to be a more reasonable characterization of household work decisions. The majority of households in the U.S. either work full time, or not at all. While part-time employment is on the rise, it is still not the norm. That is, household work hours are volatile because they are either 0 or 40 hours per week - rather than flexible (as we assumed in the baseline model).

(d) We can see that the mechanism through which technology shocks affect the labor market is very similar to what we saw in Question 2. A negative technology shock reduces labor demand (through reducing labor productivity), causing a reduction in the wage and work hours. Therefore, we will still see a positive correlation between work hours and productivity. However, the reduction in work hours will be far more volatile in the indivisible labor model. Rather than several households reducing their work hours by a small amount (as in the standard RBC model), a few households will move from full-time employment to no employment at all (indivisible labor). See Hansen (1985) for a table similar to Romer's Table 4.4 comparing the indivisible labor model to the standard RBC model and the data - Hansen's (1985) model is able to generate more reasonable volatility in work hours.

4. **Spending Shocks**

Consider the following deterministic, closed-economy version of an IS/MP/IA model:

$$\begin{aligned} IS &: Y = C(Y-T) + I(r) + G + \varepsilon_{IS} \\ MP &: r = r(Y,\pi) + \varepsilon_{MP} \\ IA &: \pi = \pi(Y-\bar{Y}) + \varepsilon_{IA} \end{aligned}$$

(a) The IS and MP curves express interest rates as a function of output. Therefore, inflation is treated as an exogenous variable on this diagram. Take the derivate of the expressions for IS and MP above, holding exogenous variables (and inflation) constant:

$$\begin{aligned} IS' &: dY = C'_Y dY + I'_r dr \\ MP &: dr = r'_Y dY \end{aligned}$$

Rewriting these expressions:

$$\begin{aligned} IS' &: (1-C'_Y)dY - I'_r dr = 0 \\ MP' &: r'_Y dY - dr = 0 \end{aligned}$$

Solving for the slope of each curve, $\frac{dr}{dY}$:

$$\begin{aligned} IS' &: \frac{dr}{dY} = \frac{\overset{(+)}{(1-C'_Y)}}{\underset{(-)}{I'_r}} < 0 \\ MP' &: \frac{dr}{dY} = \underset{(+)}{r'_Y} > 0 \end{aligned}$$

Therefore, the IS curve is downward sloping, and the MP curve is upward sloping.

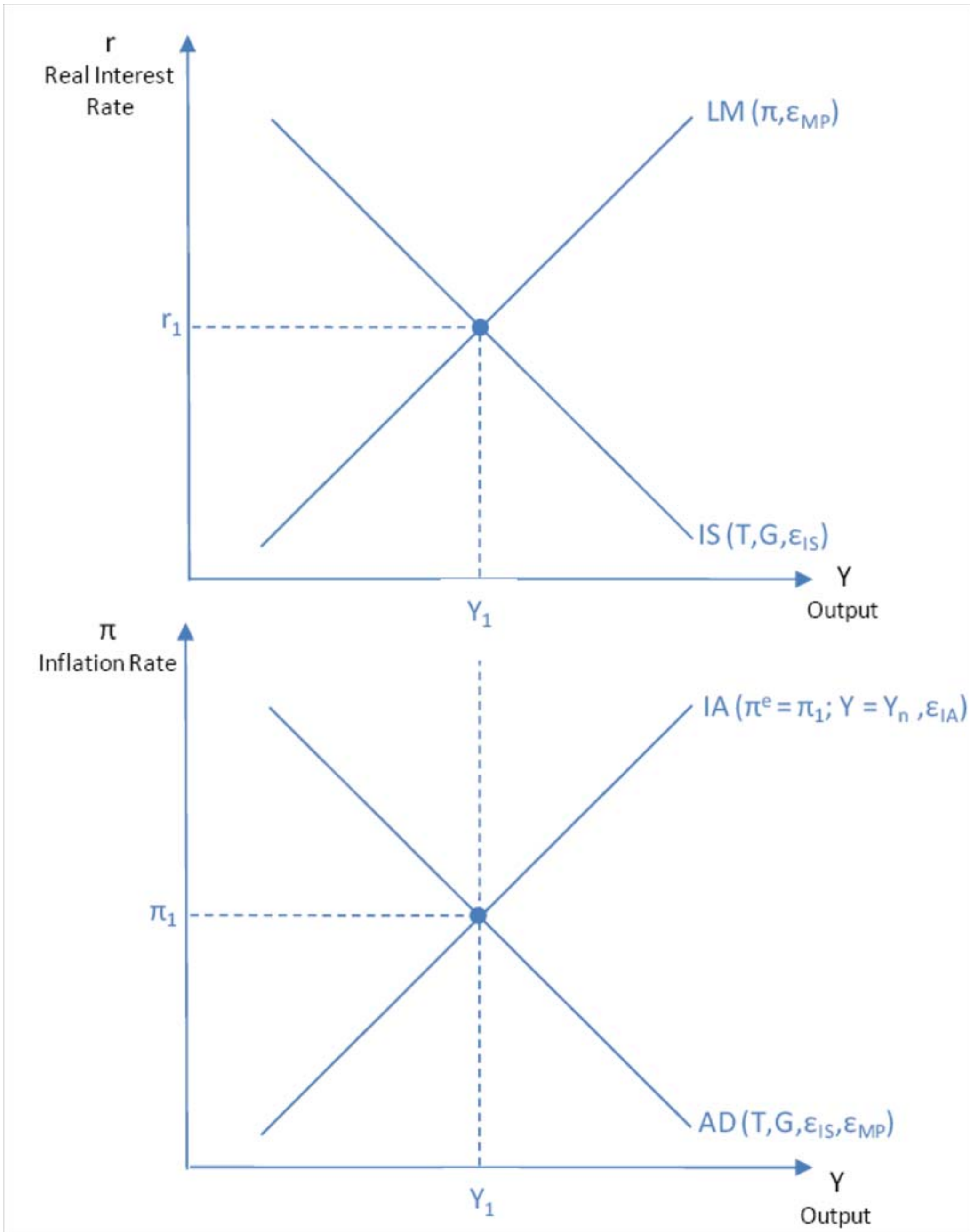
(b) The IA curve is graphed with inflation as a function of output, or $\frac{d\pi}{dY}$. Differentiating the IA curve (and holding all exogenous variables constant) we find:

$$IA' : d\pi = \pi'_Y dY$$

Rewriting:

$$\frac{d\pi}{dY} = \overset{(+)}{\pi'_Y} > 0$$

Therefore, the IA curve is upward sloping. See figure below for the IS/MP/IA model. The shifters are shown in parenthesis. Y_n is used to denote $\bar{Y}$ or full employment output.



(c) • The central bank responds only to changes in inflation and not output.
Since the the MP curve describes the central bank, we know this involves a parameter in the MP function. If the central bank does not respond to output, then this means that r does not respond to Y, so $r'_Y = 0$:

$$MP' : \frac{dr}{dY} = r'_Y = 0$$

- Inflation is relatively inelastic (with respect to output).
  This refers to the inflation adjustment process, given by IA. If the inflation rate is relatively inelastic with respect to output, this means that inflation does not adjust to change in output, so π'_Y is a small number. If $\pi'_Y = 0$, this would mean the inflation rate is perfectly inelastic with respect to inflation. From part (a), we can see how this affects the slope of the IA curve:

$$\frac{d\pi}{dY} = \downarrow \pi'_Y > 0$$

  Therefore, the slope of the IA curve is small, or closer to zero. That is, the IA curve is flatter as inflation is less elastic with respect to output.
- Investment demand is more sensitive to changes in the cost of capital.

  Since investment appears in the IS curve, this involves a parameter that affects the IS function. If investment is more senstive to changes in the cost of capital, this implies that I'_r is larger (a larger negative number). We can see how this affects the slope of the IS curve we found in part (a):

$$\frac{dr}{dY} = \frac{(1 - C'_Y)}{\uparrow I'_r} \downarrow < 0$$

  Therefore, the slope of the IS curve is smaller (a smaller negative number), and therfore the line is flatter (closer to zero).

(d) A decrease in government spending affects the IS curve. That is, $dG < 0$. To understand the effects of this change in interest rates, output, and inflation, we need to differentiate the curves with respect to the endogenous variables and allow $dG < 0$ (in parts a and b above, we held the exogenous shock constant):

$$\begin{aligned} IS' &: \ dY = C'_Y dY + I'_r dr + dG \\ MP &: \ dr = r'_Y dY + r'_\pi d\pi \\ IA' &: \ d\pi = \pi'_Y dY \end{aligned}$$

Rewriting these expressions:

$$\begin{aligned} IS' &: \ (1 - C'_Y)dY - I'_r dr = d\varepsilon_{IS} \\ MP' &: \ -r'_Y dY + dr - r'_\pi d\pi = 0 \\ IA' &: \ d\pi - \pi'_Y dY = 0 \end{aligned}$$

In matrix notation:

$$\underbrace{\begin{bmatrix} (1 - C'_Y) & -I'_r & 0 \\ -r'_Y & 1 & -r'_\pi \\ -\pi'_Y & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} dY \\ dr \\ d\pi \end{bmatrix} = \underset{B}{\begin{bmatrix} dG \\ 0 \\ 0 \end{bmatrix}}$$

The question is asking to find the effect of this shock on output, interest rates, and inflation. That is, we need to compute $\frac{dY}{dG}$, $\frac{dr}{dG}$, and $\frac{d\pi}{dG}$. First, redefine the system, dividing through by $d\varepsilon_{IS}$:

$$\underset{A}{\begin{bmatrix} (1 - C'_Y) & -I'_r & 0 \\ -r'_Y & 1 & -r'_\pi \\ -\pi'_Y & 0 & 1 \end{bmatrix}} \underset{\mathbf{x}}{\begin{bmatrix} \frac{dY}{dG} \\ \frac{dr}{dG} \\ \frac{d\pi}{dG} \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

To find these derivatives, we will use Cramer's Rule. First, we need to find the determinant of matrix A :

$$\begin{aligned}
|A| &= (1-C'_Y)\left[(1)(1)-(0)(-r'_\pi)\right]-\left(-I'_r\right)\left[(-r'_Y)(1)-(-\pi'_Y)(-r'_\pi)\right] \\
&\quad +0\left[\left(-r'_Y\right)(0)-\left(-\pi'_Y\right)(1)\right] \\
&= \underbrace{(1-C'_Y)}_{(+)}+\underset{(-)}{I'_r}\underbrace{\left[\underset{(+)}{(-r'_Y)}-\underset{(+)}{(\pi'_Y)}\underset{(+)}{(r'_\pi)}\right]}_{(+)}=(+)+(-)(+)=(+) \\
|A| &> 0
\end{aligned}$$

According to Cramer's Rule, the $\frac{dY}{dG}$, $\frac{dr}{dG}$, and $\frac{d\pi}{dG}$ are found by doing the following. If the derivative appears in the i^{th} row of the vector $\mathbf{x}$, replace the i^{th} column in the A matrix with the solution vector B and divide by the determinant of the A matrix:

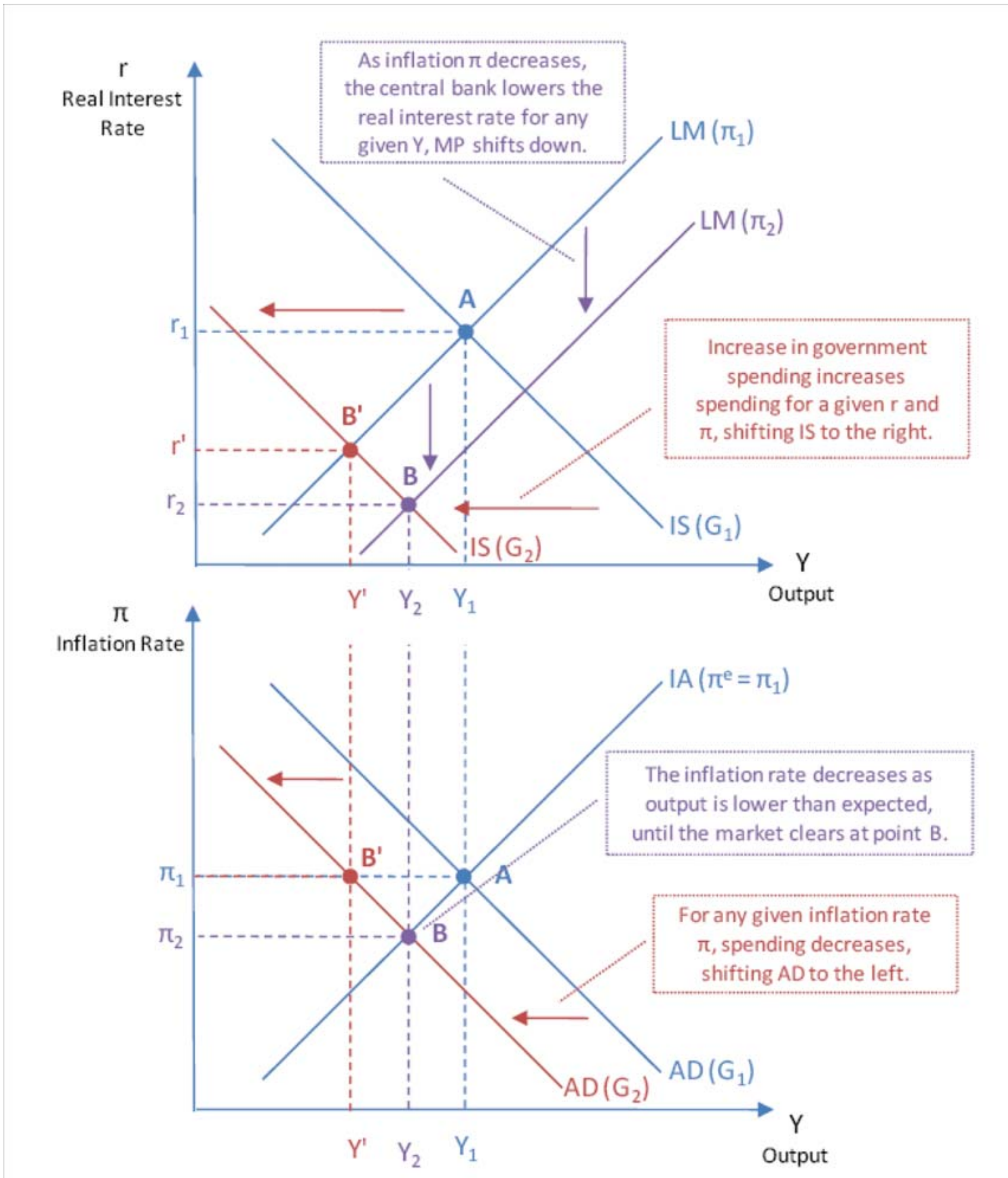
$$\frac{dY}{dG}=\frac{\begin{vmatrix}\mathbf{1} & -I'_r & 0\\ \mathbf{0} & 1 & -r'_\pi\\ \mathbf{0} & 0 & 1\end{vmatrix}}{|A|}=\frac{1\left[(1)(1)-(1)(0)\right]}{|A|}=\frac{1}{|A|}=\frac{(+)}{(+)}>0$$

$$\frac{dr}{dG}=\frac{\begin{vmatrix}(1-C'_Y) & \mathbf{1} & 0\\ -r'_Y & \mathbf{0} & -r'_\pi\\ -\pi'_Y & \mathbf{0} & 1\end{vmatrix}}{|A|}=\frac{-1\left[(-r'_Y)(1)-(-\pi'_Y)(-r'_\pi)\right]}{|A|}=\frac{\overbrace{r'_Y-\pi'_Y r'_\pi}^{(+)}}{|A|}=\frac{(+)}{(+)}>0$$

$$\frac{d\pi}{dG}=\frac{\begin{vmatrix}(1-C'_Y) & -I'_r & \mathbf{1}\\ -r'_Y & 1 & \mathbf{0}\\ -\pi'_Y & 0 & \mathbf{0}\end{vmatrix}}{|A|}=\frac{1\left[-r'_Y(0)-(-\pi'_Y)(1)\right]}{|A|}=\frac{\overset{(+)}{\pi'_Y}}{|A|}=\frac{(+)}{(+)}>0$$

Therefore, we see a decrease in government spending leads to a decrease in output, interest rates, and inflation. Since we found positive derivatives above, we know that a decrease in G leads to a decrease in the endogenous variables, Y, r, and π. We can check to see if this is the case using the IS/MP/IA diagram below.

(e) See the diagram below.



5. **Monetary Shocks**

(a) To understand the effects of this change in interest rates, output, and inflation, we need to differentiate the curves with respect to the endogenous variables and allow $d\varepsilon_{MP} < 0$:

$$\begin{aligned} IS' &: dY = C'_Y dY + I'_r dr \\ MP &: dr = r'_Y dY + r'_\pi d\pi + d\varepsilon_{MP} \\ IA' &: d\pi = \pi'_Y dY \end{aligned}$$

Rewriting these expressions:

$$\begin{aligned} IS' &: (1-C'_Y)dY - I'_r dr = +d\varepsilon_{MP} \\ MP' &: -r'_Y dY + dr - r'_\pi d\pi = 0 \\ IA' &: d\pi - \pi'_Y dY = 0 \end{aligned}$$

In matrix notation:

$$\underbrace{\begin{bmatrix} (1-C'_Y) & -I'_r & 0 \\ -r'_Y & 1 & -r'_\pi \\ -\pi'_Y & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} dY \\ dr \\ d\pi \end{bmatrix} = \underset{B}{\begin{bmatrix} 0 \\ d\varepsilon_{MP} \\ 0 \end{bmatrix}}$$

The question is asking to find the effect of this shock on output, interest rates, and inflation. That is, we need to compute $\frac{dY}{d\varepsilon_{MP}}$, $\frac{dr}{d\varepsilon_{MP}}$, and $\frac{d\pi}{d\varepsilon_{MP}}$. First, redefine the system, dividing through by $d\varepsilon_{IS}$:

$$\underbrace{\begin{bmatrix} (1-C'_Y) & -I'_r & 0 \\ -r'_Y & 1 & -r'_\pi \\ -\pi'_Y & 0 & 1 \end{bmatrix}}_{A} \underset{\mathbf{x}}{\begin{bmatrix} \frac{dY}{d\varepsilon_{MP}} \\ \frac{dr}{d\varepsilon_{MP}} \\ \frac{d\pi}{d\varepsilon_{MP}} \end{bmatrix}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

To find these derivatives, we will use Cramer's Rule. First, we need to find the determinant of matrix A :

$$\begin{aligned} |A| &= (1-C'_Y)\left[(1)(1)-(0)(-r'_\pi)\right] - \left(-I'_r\right)\left[\left(-r'_Y\right)(1) - (-\pi'_Y)(-r'_\pi)\right] \\ &\quad +0\left[\left(-r'_Y\right)(0) - \left(-\pi'_Y\right)(1)\right] \\ &= \underbrace{(1-C'_Y)}_{(+)} - \underset{(-)}{I'_r} \underbrace{\left[\underset{(+)}{r'_Y} + \underset{(+)(+)}{\pi'_Y r'_\pi}\right]}_{(+)} = (+)-(-)(+) = (+) \\ |A| &> 0 \end{aligned}$$

Notice, the A matrix is always the same, regardless of the shock affecting the economy. According to Cramer's Rule, the $\frac{dY}{d\varepsilon_{MP}}$, $\frac{dr}{d\varepsilon_{MP}}$, and $\frac{d\pi}{d\varepsilon_{MP}}$ are found by doing the following. If the derivative appears in the i^{th} row of the vector $\mathbf{x}$, replace the i^{th} column in the A matrix with the solution vector B and divide by the determinant of the A matrix:

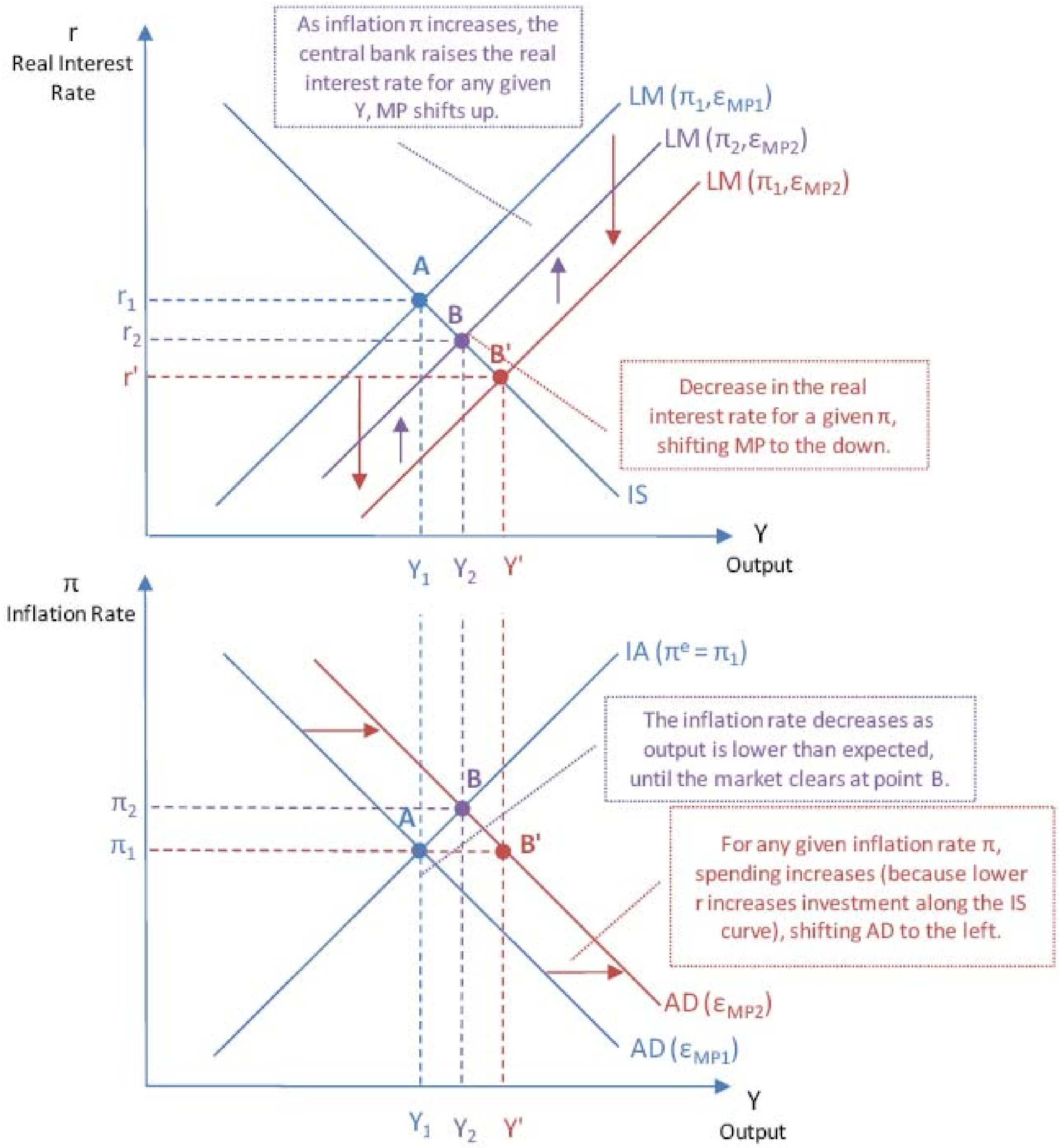
$$\frac{dY}{d\varepsilon_{MP}} = \frac{\begin{vmatrix} \mathbf{0} & -I'_r & 0 \\ \mathbf{1} & 1 & -r'_\pi \\ \mathbf{0} & 0 & 1 \end{vmatrix}}{|A|} = \frac{-1\left[\left(-I'_r\right)(1)-(0)(0)\right]}{|A|} = \frac{\overset{(-)}{I'_r}}{|A|} = \frac{(-)}{(+)} < 0$$

$$\frac{dr}{d\varepsilon_{MP}} = \frac{\begin{vmatrix} (1-C'_Y) & \mathbf{0} & 0 \\ -r'_Y & \mathbf{1} & -r'_\pi \\ -\pi'_Y & \mathbf{0} & 1 \end{vmatrix}}{|A|} = \frac{1\left[(1-C'_Y)(1)-\left(-\pi'_Y\right)(0)\right]}{|A|} = \frac{\overbrace{(1-C'_Y)}^{(+)}}{|A|} = \frac{(+)}{(+)} > 0$$

$$\frac{d\pi}{d\varepsilon_{MP}} = \frac{\begin{vmatrix} (1-C'_Y) & -I'_r & \mathbf{0} \\ -r'_Y & 1 & \mathbf{1} \\ -\pi'_Y & 0 & \mathbf{0} \end{vmatrix}}{|A|} = \frac{-1\left[(1-C'_Y)(0)-(-\pi'_Y)(-I'_r)\right]}{|A|} = \frac{\overset{(+)(-)}{\pi'_Y I'_r}}{|A|} = \frac{(-)}{(+)} < 0$$

Therefore, we see that an increase in the MP curve leads to an increase in interest rates and a reduction in output and inflation.

(b) See the diagram below. The decrease in the interest rate leads to an increase in output and inflation that causes the central bank to response endogenously by raising interest rates somewhat (but not by enough to offset the initial increase).



References

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