ECON 200A
Spring 2007

Midterm Exam
Monday, March 12^{th}

You have a total of three hours to complete this exam. Please begin each question on a separate sheet of the paper provided .

1. *Classical Production Function with Land* [15 points]
   Assume the California economy has a Cobb-Douglas production function with $Y = K^{\alpha}L^{\beta}T^{\gamma}$. Markets are perfectly competitive. Input prices are as follows: r is the rental rate for capital, w is wage, and q is the real rental rate for land. The price of output is equal to 1.

   (a) Set up the profit maximization problem and derive the first-order conditions for this problem.
   (b) Using your answer to part (b), compute the real wage w, real rental rate for capital r, and real rental rate for land, q.
   (c) Using your answers above, compute the shares of income paid to capital, labor, and land.

2. *Diamond Model* [20 points]
   Suppose that household utility is given by the following:

$$U = \log(C_{1,t}) + \frac{1}{1+\rho}\log(C_{2,t+1})$$

   Households earn real wages w_t in the first period when they are working. The households divide their wages between consumption in period 1, $C_{1,t}$ and savings. Each unit the household saves earns a real interest rate r, paid out in the second period (as r_{t+1}). In the second period, they retire, and must rely on their savings to finance consumption $C_{2,t+1}$ in this period.

   (a) Set up the household's utility maximization problem and derive the first-order conditions for this problem.
   (b) Write out the Euler equation from your utility maximization problem above.
   (c) Compute the household savings rate.
   (d) The government is considering introducing a compulsory retirement program. In this system (identical to the one we have today), each member of the young generation makes a transfer payment T_t to the old generation. Based on this information, write out a budget constraint for the individual household.
   (e) Suppose the population growth rate is currently 2%, the nominal interest rate is 5% and the inflation rate is 3%. How does this retirement program affect household savings? Explain briefly. [Note: You can show the outcome mathematically, or explain in words].

3. *Solow Growth Model* [20 points]
   Assume an extended Solow growth model for the economy with the Cobb-Douglas production function $Y = K^{\alpha}(AL)^{1-\alpha}$.

   (a) Illustrate the steady state level of capital, output, consumption, and investment (in per effective worker terms) on a Solow growth diagram.

   (b) Suppose that the share of income paid to capital is currently 30%, the depreciation rate is 2%, and the savings rate is 40%. Is the economy above or below the golden rule level of capital per effective worker? Explain briefly how you know. [Note: You don't need to derive an expression for the golden rule if you can recall the rule from class/homework].

   (c) Suppose the government implements a policy designed to achieve the golden rule. Illustrate how this policy would affect the economy, using your graph from part (a).

   (d) Using impulse responses, illustrate how output (per effective worker), consumption (per effective worker) and wages (per worker in log terms) transition to their new steady state values when this policy is implemented. The growth rate of wages is given by:

$$\frac{\dot{w}}{w} = \frac{\dot{A}}{A} + \frac{-f''(\widetilde{k})\dot{\widetilde{k}}\widetilde{k}}{f(\widetilde{k}) - f'(\widetilde{k})\widetilde{k}}$$

4. *Endogenous Growth Models* [25 points]
   This problem considers how an economy's openness affects its level of sophistication in two different growth models. In the model below, A denotes the world technology frontier.

   (a) Construct a graph with $\frac{\dot{h}}{h}$ expressed as a function of $\frac{A}{h}$. Plot the following two lines:

$$\frac{\dot{h}}{h} = \mu e^{\psi u}\left(\frac{A}{h}\right) \qquad\qquad \frac{\dot{h}}{h} = g$$

   Interpret the economic meaning of each of these two lines and discuss the importance of the point where they intersect.

   (b) Using the model above, analyze the short-run and long-run effects of a decrease in μ on the growth rate of h.

   (c) The Lucas (1988) model specifies the growth in human capital according to $\dot{h} = (1-u)h$ and the production function is: $Y = K^{\alpha}(hL)^{1-\alpha}$. Compare and contrast the endogenous growth model in part (a) to the Lucas (1988) model in terms of the following assumptions: technology/technological progress, savings, and market structure.

   (d) Suppose the government reduces its subsidies directed toward research universities. How will this likely affect u in the Lucas (1988) model? How will it affect μ in the model shown in part a)? Explain briefly.

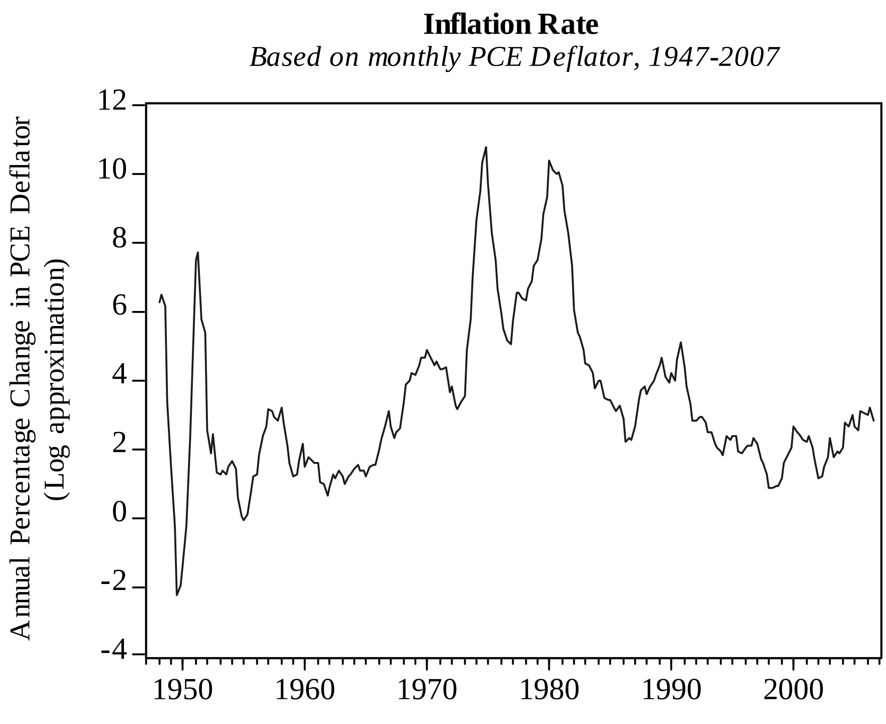
   (e) Discuss the effects of this policy for the *level* of per capita income and for the *growth rate* of per capita income. Compare and contrast the implications from each model above.

Empirical Questions

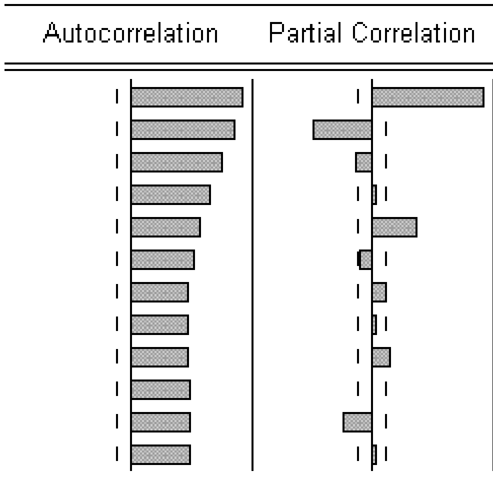
5. *Convergence* [10 points]
   In your answers below, you should cite the findings of Prichett (1997) and Romer (1994) where appropriate.

   (a) Describe an empirical test of convergence across countries. Specifically, write a cross section regression specification involving two variables from the data.
   (b) Based on the data, do countries converge in terms of their per capita income? How does your answer depend on which subsets of data under examination?

6. *Forecasting Inflation* [10 points]
   Consider the plot, correllogram, and accompanying regression results for the inflation rate. Based on the correllogram and the regression results, which $ARMA(p, q)$ is the best fit? You should justify your choice by referring to the correllogram and regression results. Please note that since these are actual data, it is important to look at several different criteria when making your decision. Be sure to clearly explaining these in justifying your choice.



Sample: 1947Q1 2007Q1
Included observations: 235



Lag	AC	PAC	Q-Stat	Prob
1	0.961	0.961	219.86	0.000
2	0.885	-0.514	406.89	0.000
3	0.786	-0.135	555.04	0.000
4	0.680	0.040	666.59	0.000
5	0.598	0.389	753.24	0.000
6	0.535	-0.100	822.95	0.000
7	0.499	0.123	883.80	0.000
8	0.487	0.040	941.95	0.000
9	0.491	0.148	1001.4	0.000
10	0.508	-0.007	1065.2	0.000
11	0.514	-0.238	1130.9	0.000
12	0.509	0.037	1195.7	0.000

	A *AR(2)*	**B** *AR(3)*	**C** *MA(2)*	**D** *ARMA(2,2)*
Constant	3.212*** (0.611)	3.260*** (0.546)	3.364*** (0.186)	3.062*** (0.878)
AR(1)	1.492*** (0.041)	1.419*** (0.066)	--	0.529*** (0.100)
AR(2)	-0.549*** (0.055)	-0.351*** (0.112)	--	0.384*** (0.095)
AR(3)	--	-0.133** (0.066)	--	--
MA(1)	--	--	1.046*** (0.041)	1.107*** (0.085)
MA(2)	--	--	0.756*** (0.039)	0.132* (0.080)
R-squared	0.951	0.951	0.823	0.955
Adjusted R-squared	0.950	0.951	0.821	0.954
AIC	1.599	1.593	2.884	1.532
SIC	1.643	1.652	2.929	1.607
Durbin-Watson	2.137**	1.931**	0.872[†]	1.791[†]
No. observations	233	233	233	233

Standard errors are reported in parentheses.
*, **, *** indicates significance at the 90%, 95%, and 99% confidence level, respectively.
** Durbin-Watson statistic rejects the presence of autocorrelation in the residuals (at the 95% confidence level).
[†] Durbin-Watson statistic fails to reject the presence of autocorrelation in the residuals.