In my visits to elementary classrooms lately I have been reminded about the art of moving students from guessing to reasoning logically. Consider the three scenarios which follow.

One Scenario
The teacher says, “I will put up some data and I would like you each to figure out the rule.”
The teacher writes the following on the board, hoping it will be enough information for the students to figure out to double the input and add 1 in order to get the output:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
</tr>
</tbody>
</table>

Then the teacher gives the students time to copy this down and to think about what rule could be applied to each input to give the corresponding output. The teacher walks around the room monitoring students, stopping and helping some individuals along the way. After a few minutes the teacher asks, “Who can give me the rule that can be used to determine the outputs from each input?” Some students raise their hands, and one student is called upon to give an answer.

What did the students learn? Some students had already developed strategies for determining rules like this and were able to figure out the rule independently. These questions always seemed to sort the class into two groups—those who got it and those who didn’t.

Another Scenario
The teacher says, “Who can give me a number less than 20 to use for an input?” Some students raise their hands and the teacher calls on one to give an input—perhaps 5.
The teacher writes on the following on the board:

...continued on page 2.
“Just Guessing?”
...continued from page 1.

The teacher says, “I am thinking of a rule to apply to the input in order to get the output. Who is willing to take a guess at what will come out if a 5 goes in?” Some hands go up and the teacher calls on one student—perhaps the student says 18. The teacher says, “Great guess, the tens digit of your guess is the same as the tens digit of the output. Who would like to make another guess?” The next student might say 15. The teacher is constantly thinking about a way to use the students’ guesses to provide clues as to the correct number for the input. The teacher responds, “You are getting closer. You too have the correct tens digit, and the output is only 4 smaller than your guess.” The teacher then says to the class, “How many students have a guess about what the output is if the input is 5?” Most students have their hands up. “Who would like to share a guess?” Someone at this point will have used the clues to determine the output is 11, at which point the teacher will write this on the board, and then provide another input. The board will look like:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

Then the teacher asks, “Who has a guess about the output if the input is 3?” The students have figured out that no matter what their guesses are, the teacher will use their guesses to provide clues about what the corresponding output values will be.

What did the students learn? The students have determined that the key to determining outputs is to keep guessing. The teacher will use each guess, no matter how far off it is, to provide clues as to what the output should be. The students like this guessing game and listen carefully to the clues provided by the teacher. Eventually some students figure out the rule actually being applied by the teacher, because enough input and output pairs have been provided. This will also sort the class into two groups, those who get the rule and those who don’t, just like the first scenario, however the students who don’t get the rule will still be involved in the guessing game.

Another third scenario begins the same as the previous scenario where the teacher gets the first input value from a student, elicits a guess at the output, and uses the student guess of 18 to provide a clue about the tens digit. The differences in this scenario begin after this and are italicized.

The teacher says, “Great guess, the tens digit of your guess is the same as the tens digit of the output that I was thinking of, because the rule is making the input larger. Who would like to make another guess?” The next student might say 15. The teacher is constantly thinking about a way to use the student’s guess to provide clues as to the correct number for the input, and thinking about how to drop clues that the students should be looking for some way to look at the input and figure out the correct output. The teacher responds, “Your are getting closer. You too have the correct tens digit, and the output is only 4 smaller than your guess.” The teacher then says to the class, “How many students have a guess about what number comes out if a 5 goes in?” Most students have their hands up. “Who would like to share a guess?” Someone at this point will have used the clues to determine the output is 11, at which point the teacher will write this on the board, and then provide another input. The board will look like:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

The teacher says, “Raise your hand if you have a possible rule that could be applied to an input of 5 to get an output of 11 out.” After surveying the hands, “I wonder if you all are thinking of the same rule as I am? How many people think it would be helpful to see this rule work for some more input values?” Hands are raised, so the teacher either chooses another input value or elicits one from a student.

Suppose the next input value is 3. The teacher asks, “How many people think that since this input is smaller than 5, the resulting output is likely to be smaller than 11?” Waits for hands. “Who would like to take a guess at what the output will be if the input is 3?” The teacher uses the student guesses to provide clues about the correct output value until getting some student (or many) to say 7. Then the teacher writes the following on the board:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

In order to prompt the students to think about looking for the rule, the teacher asks, “I wonder if you can find a way to look at the input value and figure out the output value? How many people are starting to figure out the secret to this rule, so that you can look at the input and figure out the output?” At this point, especially since this rule is a difficult one especially since this rule is difficult since it requires two operations, the teacher might drop clues about needing two operations.

What did the students learn? As in the previous scenario the students learned that the teacher will use their guesses to provide clues about the correct outputs. Moreover, the students were pointed in the direction of determining what the rule was when they were asked if they could figure out a way to determine the output by looking at the input. In the previous scenario, students became so enthralled with just guessing outputs (almost randomly without any association to the corresponding inputs) and then using clues from the teacher to refine their guesses, that they didn’t want to determine the rule being applied. Instead, in the third scenario, the teacher focused the students on trying to figure out the rule—the secret to determining the output once the input was known. By asking for guesses in conjunction with hinting there was a way to figure it out, the teacher helped to get the students to want to know the rule.

What did I learn as a teacher? Students will guess when they know there is not penalty for being wrong. Guessing can be a fun way to get students involved. In order to get them to think after guessing, we need to provide focused clues or questions to get them to look for the mathematical logic before them.

...continued on page 3.
“Just Guessing?” 
...continued from page 2.

In the scenarios above, the teacher might do this by dropping more clues about the function:
- How many people think it is not likely that subtraction will be involved in this rule since the output is larger than the input?
- How many people think this is a tricky rule? It is tricky because it uses more than one operation.
- How many people are really good at the tens multiplication facts? Okay, so how many people think we should use an input of 10 next?
- How many people are starting to figure out this rule? Okay, so I would like you to think in your mind what the output would be if the input is 7. Now whisper to your neighbor what you think the output would be.

These questions might help students that didn’t yet see the logic behind the rule, to figure out the rule for themselves. The students that don’t get the rule immediately are given more opportunities to continue trying to figure it out, instead of hearing an answer relatively quickly as in the first scenario. The students also get involved in the lesson through guessing outputs (which is a less risky endeavor for a learner and one she/he is more likely to try), and are then invited to determine the secret of the rule being applied. Their initial random guesses lead to a desire to understand the secret of the rule being applied. They report that after playing the progression of games for several months, the children were given a test with 100 multiplication problems to complete in 10 minutes. All children but one got 100% of the answers correct within the 10 minutes. (One child missed 2 questions.)

Any teacher is welcome at any of our follow-up Saturday meetings. We always begin the day learning some mathematics. After the morning lesson a public debriefing of the morning lesson is lead by observers of the lesson and the instructors of the lesson. Following this discussion we offer break-out sessions for teachers of different grade levels looking at the toughest things to teach for understanding. We wrap up the day with an opportunity to hone your abilities to observe subtleties in the classroom and to provide supportive, insightful feedback by watching video of a classroom teacher. We will feed you morning munchies and lunch. If you would like to attend, even if you have not been able to attend one of our summer institutes, please call our Administrative Assistant, Huong Nuygen at (916) 278 - 4785 or email csusmp@csus.edu with your name and the school where you teach. We ask that everyone RSVP so that we can be sure to order the right amount of food.

An important note: For those of you interested in Algebra, Professor Gary Shannon, instructor for our Algebra Institutes, will be offering a break-out session at the November meeting.

Also, remember to set aside Saturday, March 6th for our special Saturday Meeting combined with the Sacramento Area Mathematics Educators (SAME) Conference to be held in the University Union at CSUS. CSUSMP is sponsoring this event. Flyers and registration forms will be mailed out to you in January. Be on the look out!

Check It Out!

I know we all have too much to read but...The November issue of Teaching Children Mathematics has an article by Constance Kamii and Catherine Anderson. They share some games they used in Ms. Anderson's third grade class at a Title I school to build students knowledge, understanding and speed with the multiplication facts to 12. They report that after playing the progression of games for several months, the children were given a test with 100 multiplication problems to complete in 10 minutes. All children but one got 100% of the answers correct within the 10 minutes. (One child missed 2 questions.)

Unravel Mathematics 
CMC Asilomar Conference

Friday, December 5 - 
Sunday, December 7, 2003

If you haven't been to this conference, find a way to go. If offers a wide variety of workshops, talks and interactive discussion of issues in a beautiful setting at the Asilomar Conference Grounds in Pacific Grove. Many of you have heard Debbie say that the first time she attended, she was so moved to know that she was at a conference with so many other teachers trying to make their teaching better for their students. The many ideas you will hear, the kindred spirits in attendance, and the fresh open surroundings will inspire you to reflect about new things to try with your class. You may be able to find financial assistance in your school district as there may be professional development monies available to help teachers attend conferences aimed at improving mathematics education. CSUSMP fellows will find it a great place to reconnect. If you would like more information about the conference, please go to: http://www.cmc-math.org/ASIL.
Mistakes are the portals of discovery.

—James Joyce

CSUS Math Project News - Fall 2003 Edition

“Mistakes are the portals of discovery.”