

California State University
Department of Mathematics and Statistics
SYLLABUS

Math 101– Combinatorics (3 units)

Prerequisites: Math 31 Calculus II.

Introduction to the art of counting. The focus will be on actually listing the objects being counted in small cases and using the knowledge gained in working with small cases to build toward general principles. Sum and product principles, models of counting, permutations and combinations, equivalence relations and partitions, inclusion-exclusion principle, recurrence relations, and generating functions.

OUTLINE:

I. Introduction (2 weeks)

- a. Sum and product principles
- b. Ordered pairs, relations, and functions
- c. Subsets
- d. Binomial and multinomial theorems
- e. Mathematical Induction
- f. Pigeonhole principle

II. Models of counting (2 weeks)

- a. Selection (n elements to chose from, select k of them)
 - i. k -tuples (with replacement, order important)(functions)
 - ii. k -tuples (without replacement, order important) (permutations) (one-to-one functions)
 - iii. k -sets (without replacement, order not important)(combinations)
 - iv. k -multisets (with replacement, order not importatn)
- b. Distributuions (n distinguishable boxes and k balls to put in them)
 - i. Balls distinguishable and boses allowed to contain more than one ball
 - ii. Balls distinguishable and boses allowed to contain at most one ball
 - iii. Balls indistinguishable and boses allowed to contain at most one ball
 - iv. Balls indistinguishable and boses allowed to contain more than one ball

- III. Equivalence relations (1 week)
 - a. Circular arguments
 - b. Equivalence classes
 - c. Correspondence and partitions
 - d. Inverses
- IV. Partitions (3 weeks)
 - a. Set partitions
 - b. Stirling numbers
 - c. Integer partitions
 - d. Ferrers and Young diagrams
 - e. Conjugate of a partition
- V. Inclusion-exclusion principle (2 weeks)
 - a. Two and three sets
 - b. Arbitrary number of sets
 - c. Onto functions
 - d. Derangements and the Menage problem (one-to-one functions where no element is mapped to itself.)
- VI. Introduction to recurrence relations (2 weeks)
 - a. Relevance
 - b. Fibonacci numbers
 - c. Second order linear recurrence relations
 - d. Partial fractions
 - e. Catalan numbers
- VII. Introduction to generating functions (2 weeks)
 - a. Power series
 - b. Product principle
 - c. Polynomial generating functions
 - d. Extended binomial theorem