

CALIFORNIA STATE UNIVERSITY, SACRAMENTO

Department of Mathematics and Statistics

SYLLABUS

Math 110A and 110B: Modern Algebra
Prerequisites: Math 110A – Math 108
Math 110B–Math 110A

This is a one year introductory course in Modern Algebra. It is a required course for all students majoring in mathematics. An introduction to groups, rings, fields, and vector spaces will be presented. Some applications may be selected from cryptography, algebraic coding theory, software design, Boolean algebras, electrical circuits, quaternions, finite fields, and constructible numbers, as time permits.

Math 110A

- I. Basic properties of groups. 3 weeks
- a. Binary operations.
 - b. Identities and inverses.
 - c. Group tables.
 - d. Subgroups.
 - e. Examples chosen from \mathbb{Z} , \mathbb{Q} , \mathbb{R} , \mathbb{C} , $M_{m \times n}(\mathbb{R})$, and \mathbb{Z}_n , under addition, and \mathbb{Q}^* , \mathbb{R}^* , \mathbb{C}^* , $GL_n(\mathbb{R})$, $\mathbb{Z}_n^* = \{\bar{a} \in \mathbb{Z}_n \mid (a, n) = 1\}$, $U = \{z \in \mathbb{C}^* \mid |z| = 1\}$, and $U_n = \{z \in \mathbb{C}^* \mid z^n = 1\}$, under multiplication.
- II. Cyclic groups and their subgroups. 2 weeks
- a. Classification of cyclic groups.
 - b. Subgroups of finite cyclic groups.
 - c. Subgroup lattice diagrams for \mathbb{Z}_n .
- III. Permutation groups. 2 weeks
- a. Cayley's theorem.
 - b. Orbits, cycles, and cycle decompositions.
 - c. Even and odd permutations and A_n .
 - d. Dihedral groups.
- IV. Cosets. 2 weeks
- a. Theorem of Lagrange.
 - b. The index of a subgroup.

- V. Direct Products. 1 week
- a. Fundamental theorem of finite abelian groups (without proof).
- VI. Group homomorphisms. 4 weeks
- a. The kernel of a homomorphism.
 - b. Normal subgroups.
 - c. Factor groups.
 - d. Fundamental theorem of group homomorphisms.
- Math 110B
- I. Basic properties of rings. 4 weeks
- a. Definition, examples, and elementary properties.
 - b. Subrings.
 - c. Integral domains, zero divisors, and cancellation.
 - d. Multiplicative inverses and fields.
 - e. Characteristic of a ring.
 - f. Field of quotients of an integral domain.
- II. Ring of polynomials. 2 weeks
- a. The division algorithm in $F[x]$, with F a field.
 - b. Irreducibility in $F[x]$ and the factor theorem.
 - c. Results from $\mathbb{Q}[x]$: Rational root theorem, factoring in $\mathbb{Q}[x]$ and $\mathbb{Z}[x]$, Eisenstein criterion.
- III. Ring homomorphisms. 3 weeks
- a. Ideals and factor rings.
 - b. Fundamental theorem of ring homomorphisms.
 - c. Maximal and prime ideals.
 - d. Ideal structure in $F[x]$ and principal ideals.
- IV. Vector spaces over a field. 2 weeks
- a. Definitions and examples.
 - b. Linear dependence and independence.
 - c. Basis and dimension.
- V. Extension Fields. 3 weeks
- a. Kronecker's theorem.
 - b. Algebraic and transcendental elements.
 - c. Simple extensions.
 - d. The Fundamental Theorem of Algebra (without proof).