CALIFORNIA STATE UNIVERSITY, SACRAMENTO
Department of Mathematics and Statistics

SYLLABUS

Math 230A-B: Real Analysis

Prerequisite: Math 130B
Math 230A is a prerequisite to Math 230B.
Sequence begins every other Fall.

Numbers and sets; metric topology; sequences and series of constants and functions; continuous functions; the theory of the derivative; the theory of the integral, including Riemann, Riemann-Stieltjes, and Lebesgue integrals; measure theory on the real line.

OUTLINE: Math 230A

I. Number and sets

a. Axioms for the real number system
b. Algebra of sets
c. Functions
d. Finite, countable and uncountable sets
e. Products of sets and the axiom of choice

II. Topology of Metric Spaces

a. Open and closed sets
b. Compact sets
c. Bolzano-Weierstrass theorem
d. Connected sets
e. Perfect sets and the Cantor set

III. Sequences and Series

a. Convergence and divergence
b. Algebraic properties
c. Bounded monotone sequences
d. Subsequences
e. Cauchy criterion
f. Completeness and Baire category
g. Lim sup and lim inf
h. Root and ratio test
i. Absolute and conditional convergence

IV. Limits and Continuity

a. Algebraic properties
b. Types of discontinuities and monotone functions
c. Continuity and compactness
d. Uniform continuity
e. Absolute continuity

V. Differentiation

a. Algebraic properties
b. Continuity and derivatives
c. Continuous, nowhere differentiable function
d. Intermediate value theorem
e. Local extrema
f. Mean value theorems
g. L'Hospital's rule
h. One-side derivatives

OUTLINE: Math 230B

I. Riemann-Stieltjes Integration

a. Monotone functions and bounded variation
b. The Riemann-Stieltjes integral as a generalization of the Riemann integral
c. Mean value theorems
d. Conditions for integrability
e. Reduction to Riemann integral
f. Fundamental Theorem of Calculus

II. Sequences and Series of Functions

a. Pointwise, uniform and Cauchy convergence
b. Continuity and uniform convergence
c. Differentiation and uniform convergence
d. Integration and uniform convergence
e. Equicontinuity
f. Stone-Weierstrass theorem
g. Power series and Taylor series

III. Measure on the Real Line

a. Inner and outer measure
b. Measureable sets
c. Set theoretic properties
d. Translation invariance
e. Example of a nonmeasurable set
f. Vitali covering theorem

IV. Measureable functions

a. Continuity and measurability
b. Algebraic properties
c. Pointwise limits
d. Egorov's theorem
e. Lusin's theorem
f. Approximation by simple functions

V. Lebesgue Integration

a. Algebraic properties
b. Conditions for integrability
c. Convergence theorems
d. The fundamental theorem of Calculus
e. Change of variable
f. Mean value theorems

The written exam in Real Analysis will cover the content of sections I-V of Math 230A and sections I-II of math 230B

REFERENCES: Rudin - Principles of Mathematics
Bartle - The Elements of Real Analysis
Goldberg - Methods of Real Analysis