CALIFORNIA STATE UNIVERSITY, SACRAMENTO
Department of Mathematics and Statistics

SYLLABUS

Math 241A: Foundations of Applied Mathematics

Prerequisite: Math 134 is recommended.

This course covers elementary topics in metric topology and functional analysis designed to give students a good foundation for further study in modern applied mathematics. It presents an introduction to metric spaces, Banach spaces, Hilbert spaces, operators on Hilbert spaces, with applications to differential equations. This course is offered in the Fall.

COURSE OUTLINE:

I. Review of Linear Algebra and Analysis (1 week)

II. Metric Spaces (3 weeks)

- A. Metric Topology
- B. Continuity
- C. Convergence and Completeness
- D. Compactness
- E. Arzela-Ascoli Theorem
- F. Peano's Existence Theorem for Ordinary Differential Equations
  (Students will learn about convergence and completeness in metric spaces and how to apply these concepts to differential equations.)

III. Elements of Banach Spaces (2 weeks)

- A. Contraction Mapping Theorem
- B. Existence and Uniqueness Theorem for Ordinary Differential Equations
  (Students will learn the elements of Banach spaces and an application of the contraction mapping theorem to proving the existence and uniqueness of solutions of differential equations.)

IV. Elements of Hilbert Spaces (4 weeks)

- A. Orthogonal Expansions
- B. Theorems of Bessel, Parseval, and Riesz-Fischer

C. Linear Functionals and the Riesz Representation Theorem
D. Weak Convergence
(Students will learn the elements of Hilbert spaces and its application to the Fourier series expansions of functions.)

V. Operators on a Hilbert Space (4 weeks)

A. Self-Adjoint Operators
B. Spectral Theorem
C. Sturm-Liouville Theory
D. Partial Differential Equations and Separation of Variables
(Students will learn the elements of operators on a Hilbert space and their applications to solving boundary value problems.)

Reference Book: A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science*, 1982, Springer-Verlag

Math 241B: Topics in Applied Mathematics

Prerequisites: Math 241B.

With the permission of the department, this course (offered with different topics) may be taken for credit more than once. This course is offered in the Spring.

This is a topics in modern applied mathematics course designed to present a variety of methods of modern analysis and differential equations useful in solving problems originating in science and engineering. The topics, chosen from modern fields of applied mathematics, may vary depending upon the interests of the instructor. Topics may be selected from the following areas: ***Generalized Functions*** (Fourier Transforms, Green's Functions), ***Calculus of Variations*** (Euler-Lagrange Equations, Isoperimetric Inequality, Eigenvalue Problems), ***Topics in Functional Analysis*** (Hahn-Banach Theorem, Bounded and Continuous Operators, Weak* Convergence, Sobolev Spaces), ***Dynamical Systems*** (Autonomous and Nonautonomous Systems, Stability, Liapunov Functions, Poincare-Bendixson Theorem), ***Integral Equations*** (Hilbert-Schmidt Theory, Fredholm Theory), ***Potential Theory*** (Harmonic Functions, Green's Functions), and ***Optimal Control Theory*** (Controllability, Bang-Bang Principle, Pontryagin Maximum Principle).

SAMPLE COURSE OUTLINE:

I. Review of Advanced Calculus and Analysis (1 week)

II. Calculus of Variations: The Elements (2 weeks)

A. Function Spaces and Functionals
B. The Gateaux Differential
C. Euler-Lagrange Equations
(Students will learn the theoretical underpinnings of variational methods.)

III. Calculus of Variations: Further Developments and Applications (3weeks)

A. Second Variation
B. Hamilton's Principle
C. Brachistochrone Problem, Isoperimetric Inequality, and Geodesics on Surfaces
D. Variational methods applied to eigenvalue problems
(Students will apply variational methods to solve some classic optimization problems in physics and applied mathematics.)

IV. Dynamical Systems: Linear Theory (2 1/2 weeks)

A. Fundamental Solutions
B. Autonomous Linear Systems and Phase Portraits
C. Critical Points and Stability
(Students will learn to solve linear systems of differential equations, to locate the critical points of the system, and analyze the stability of the system near a critical point.)

V. Dynamical Systems: Nonlinear Theory (3 1/2 weeks)

A. Autonomous Nonlinear System
B. Linearizing a Nonlinear System at a Critical Point
C. Stability
D. Liapunov Functions
E. Poincare-Bendixson Theorem
(Students will learn to analyze the stability of a nonlinear dynamical system using both linearization and Liapunov functions, and how to apply the Poincare-Bendixson theorem to determine the existence of a closed orbit.)

VI. Dynamical Systems: Applications (2weeks)

A. Conservative Force Fields and Elliptical Planetary Orbits
B. Hamiltonian Mechanics
C. Volterra-Lotka Predator-Prey Equations
(Students will apply techniques of dynamical systems to analyze solutions of problems from applied mathematics and related fields.)

Reference Books: H. Sagan, *Introduction to the Calculus of Variations,* 1992, Dover Publications, and M. W. Hirsch and S. Smale, *Differential Equations, Dynamical Equations, and Linear Algebra,* 1974, Academic Press

Additional Reference Books for Math 241A and B:

1. R. Abraham, J. E. Marsden, and T. Ratiu, *Manifolds, Tensor Analysis, and Applications, Second Edition,* 1988, Springer-Verlag
2. V. I. Arnold, *Mathematical Methods of Classical Mechanics, Second Edition,* 1989, Springer-Verlag
3. V. I. Arnold, *Ordinary Differential Equations,* 1992, Springer-Verlag
4. S. K. Berberian, *Introduction to Hilbert Space,* 1976, Chelsea Publishing Company
5. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations,* 1989, Springer-Verlag
6. J. B. Conway, *A Course in Functional Analysis, Second Edition,* 1997, Springer-Verlag
7. R. Courant and D. Hilbert, *Methods of Mathematical Physics, Vols. I & II,* 1989, John Wiley & Sons
8. P. Dennery and A. Krzywicki, *Mathematics for Physicists,* 1996, Dover Publications
9. J. W. Dettman, *Mathematical Methods in Physics and Engineering,* 1988, Dover Publications
10. G. B. Folland, *Fourier Analysis and Its Applications,* 1992, Brooks/Cole Publishing Company
11. G. B. Folland, *Introduction to Partial Differential Equations, Second Edition,* 1995, Princeton University Press
12. I. M. Gelfand and S. V. Fomin, *Calculus of Variations,* 2000, Dover Publications
13. C. Goffman and G. Pedrick, *First Course in Functional Analysis, Second Edition,* 1983, Chelsea Publishing Company
14. D. H. Griffel, *Applied Functional Analysis,* 2002, Dover Publications
15. R. Grimshaw, *Nonlinear Ordinary Differential Equations,* 1990, Blackwell Scientific Publications
16. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields,* 1983m Springer-Verlag
17. R. B. Guenther and J. W. Lee, *Partial Differential Equations of Mathematical Physics and Integral Equations,* 1996, Dover Publications
18. K. E. Gustafson, *Introduction to Partial Differential Equations and Hilbert Space Methods, Third Edition,* 1999, Dover Publications
19. J. K. Hale, *Ordinary Differential Equations,*1969, John Wiley & Sons
20. J. Hale and H. Kocak, *Dynamics and Bifurcations,* 1991, Springer-Verlag
21. M. W. Hirsch and S. Smale, *Differential Equations, Dynamical Equations, and Linear Algebra,* 1974, Academic Press
22. S. S. Holland, Jr., *Applied Analysis by the Hilbert Space Method,* 1990, Marcel Dekker
23. W. Hurewicz, *Lectures on Ordinary Defferential Equations,* 1990, Dover Publications
24. O. D. Kellogg, *Foundations of Potential Theory,* 1953, Dover Publications

25. A. N. Kolmogorov and S. V. Fomin, *Elements of the Theory of Functions and Functional Analysis,* 1999, Dover Publications
26. A. N. Kolmogorov and S. V. Fomin, *Introductory Real Analysis,* 1975, Dover Publications
27. E. Kreyszig, *Introductory Functional Analysis with Applications,* 1989, John Wiley & Sons
28. E. R. Lorch, *Spectral Theory,* 1962, Oxford University Press
29. D. G. Luenberger, *Optimization by Vector Space Methods,* 1969, John Wiley & Sons
30. J. Macki and A. Strauss, *Introduction to Optimal Control Theory,* 1982, Springer-Verlag
31. G. L. Naber, *Topology, Geometry, and Gauge Fields: Foundations,* 1997, Springer-Verlag
32. A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science,* 1982, Springer-Verlag
33. A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations,* 1983, Springer-Verlag
34. F. Riesz and B. Sz.-Nagy, *Functional Analysis,* 1990, Dover Publications
35. H. L. Royden, *Real Analysis, Second Edition,* 1968, The Macmillan Company
36. W. Rudin, *Functional Analysis, Second Edition,* 1991, McGraw-Hill
37. W. Rudin, *Principles of Mathematical Analysis, Third Edition,* 1976 McGraw-Hill
38. W. Rudin, *Real and Complex Analysis, Third Edition,* 1987, McGraw-Hill
39. H. Sagan, *Boundary and Eigenvalue Problems in Mathematical Physics,* 1989, Dover Publications
40. H. Sagan, *Introduction to the Calculus of Variations,* 1992, Dover Publications
41. D. A. Sanchez, *Ordinary Differential Equations and Stability Theory: An Introduction,* 1968, W. H. Freeman and Company
42. L. Schwartz, *Mathematics for the Physical Sciences,* 1966, Addison-Wesley Publishing Company
43. G. F. Simmons, *Differential Equations with Applications and Historical Notes,* 1972, McGraw-Hill
44. G. F. Simmons, *Introduction to Topology and Modern Analysis,* 1963, McGraw-Hill
45. I. Stakgold, *Green's Functions and Boundary Value Problems,* 1979, John Wiley & Sons
46. G. P. Tolstov, *Fourier Series,* 1976, Dover Publications
47. F. Verhulst, *Nonlinear Differential Equations and Dynamical Systems, Second Edition,* 1996, Springer-Verlag
48. H. F. Weinberger, *A First Course in Partial Differential Equations with Complex Variables and Transform Methods,* 1995, Dover Publications
49. R. Weinstock, *Calculus of Variations With Applications to Physics and Engineering,* 1974, Dover Publications
50. E. Zauderer, *Partial Differential Equations of Applied Mathematics, Second Edition,* 1989, John Wiley & Sons