Introduction

The chapter on quadratics is a difficult topic for our students for a variety of reasons. This topic is often taught right after linear function. Historically, connections existed between linear and quadratics, but these connections have become lost or forgotten. Dewey (1900) spoke of curriculum as a map that, “serves as a guide to future experience; it gives direction; it facilitates control; it economizes effort, preventing useless wandering, and pointing out the paths which lead most quickly and most certainly to a desired results” (p. 198). Curriculum tries to remove some of these obstacles to make it easier for the students, but can also create new obstacles.

Linear function can be thought of as any function that has a constant growth or rate such as making $10 per hour or driving 75 miles per hour. Questions that can be asked would be how long would it take me to make $50? Or how many hours would it take me to get to Los Angeles from Sacramento? The algorithm for solving linear equations involves the four basic arithmetic operations of addition, subtraction, multiplication, and division. The graphing of linear equations uses the starting point as the rate to find other values. For this example, if I had $40 saved and I was saving $10 per week, then my starting point or y-intercept would be $40 and for each consecutive week I would have $50, $60, $70, and so on. The equation for this scenario is $s = 10w + 40$, where $s$ is the money saved and $w$ is the number of weeks.

The scenario for quadratics is a bit more difficult to create. When we talk about rocket science, a quadratic equation is used to model the flight path. The area of a rectangle can also incorporate quadratics. Since contextual situations are difficult to find in textbooks, the authors of these textbooks focus more on symbolic manipulation or procedures and less on the conceptual underpinning of quadratics. When I talk to the mathematics lecturers about this chapter, the common responses are that students can’t factor, use the quadratic formula, and/or can’t tell the difference between factoring and solving. This last statement is very telling, because what it means is that students cannot distinguish between a procedure and how to use the procedure to find the solution. Both are important, but how do we make this evident to our students? Focusing only on the procedures makes mathematics difficult because students can’t connect them to anything else they learn and makes students believe that procedures and mathematics are one in the same (Stigler and Hiebert 1999).

I first became aware of this when I was a supervisor of student teachers. A student teacher had called the night before to talk about the lesson she was thinking of teaching on the day I was scheduled to observe her. She told me that she was going to teach her students how to graph a quadratic equation. She stated that she was going to give them the formula for the line of symmetry which was $x = \frac{-b}{2a}$, where $b$ was the linear coefficient and $a$ was the quadratic coefficient. I asked her if she understood how to derive the formula.

After a long pause, she said no. I had never thought about this formula but was able gave her two ways to derive it. One approach was to look at the quadratic formula and the average of the two roots would give us the vertex formula. The second approach was to take the quadratic equation in the form of
and take out a factor x for the quadratic and linear term, thus we would have
\[ y = x(ax + b) + c. \]
We can get two values for x that is equal to c. These two values are \( x = 0 \) and
\[ x = -\frac{b}{a} \] and the average of these two values would also give us the vertex formula. She seemed excited
because I had helped her build her understanding of quadratics.

However, when I observed her class, she gave the students the formula without trying to build their understanding. She taught them the procedure of substituting the values into the formula to get line of symmetry. I wasn’t shocked, I was asking her to change her way of teaching and thinking about mathematics. It would have been unfair to expect her to learn how to teach differently than how she was taught and was teaching without providing the structure and support she needed to reflect on her teaching and how to teach in a different manner.

I realized that in order to teach quadratics with conceptual understanding that I needed to connect it with linear function. More connections to prior knowledge will build a stronger understanding. I scoured my textbooks but found no lessons on how to tie the two topics together (Dolciani, Wooton et al. 1967; Foster, Winters et al. 1998; Larson, Boswell et al. 2004). I also read books about the history of algebra to see how to find these connections (Bashmakova and Smirnova 2000). I grew worried because without this connections all of conceptual work with linear would be of no value and wasted because I would have to go back to a procedural approach toward mathematics.

For those CSU-students who needed two semesters of remediation, some stayed with me for both semesters, which allowed me to develop conceptually and procedurally linear and quadratic functions with them. But for those others students, they would enroll in other classes that had a more procedural approach to mathematics. Teach the students to think critically for one semester and then teach them procedurally for the next semester made absolutely no sense. So for this submission, I will focus on one aspect of quadratics which is the graphing of quadratic equations different from a traditional approach.

1. **What question or issue were you addressing with this activity?**

Is it possible to infuse a conceptual approach to graphing into developmental mathematics classes? And what would problem solving look like in an assessment of student learning?

2. **What data did you collect to address this question or issue?**

For this past academic year, I have spoken to my lecturers about a different way of graphing quadratics and hoped that they would incorporate it into their classrooms. I asked them to collect artifacts such as assessments from the students. I then asked them how the lesson went and students’ reactions to this alternate approach to graphing. Finally, I interviewed a lecturer about his reflection on using this alternate approach.

3. **What did the data tell you?**

I knew that asking the lectures to change their approach wouldn’t be easy. There is a level of discomfort in trying something new. All of the lectures stated that they would try and below are two tasks taken from
assessments from two different lecturers. Most of them said that they taught the students both methods and allowed the students to decide their own approach. Figure 1 shows the work of a student on a task.

Figure 1. Graphing a quadratic equation using the traditional approach.

For this student above, the student applied the formula for the line of symmetry, \( x = \frac{-b}{2a} \) (see 1 above)

The student used the \( x = -4 \) to find the \( y \)-value (see 2). The student found the \( y \)-intercept (see 3) and used these values to graph the equation (see 4).

Figure 2 is the work of student using the alternate approach to graphing.

Figure 2. Graphing quadratics using an alternate approach.
This approach doesn’t require knowing the formula but understanding how to use factoring and applying that to find the vertex. The student factored to find two coordinates (see 1 above). With these two coordinates, the student found the x-coordinate for the vertex and the corresponding y-coordinate (see 2) by applying the midpoint formula. The student then picked another x-value for the fourth point (see 3) and placed these points into a table of values (see 4). With these values, the student plotted these points into the graph (see 5).

The traditional approach utilized the vertex formula but made no connection to the midpoint formula. Students did practice the skill for evaluating a formula and equation. The alternate approach used factoring, the midpoint formula, tables, and the skill for evaluating an equation. The alternate approach interweaved these different mathematical components to achieve a goal. This interweaving was important because it allowed students to develop procedural fluency for a procedure and how it connected to other procedures.

I interviewed a staff member and he remarked that those students who remembered how to graph gravitated toward the traditional approach. The weaker students who didn’t remember chose to graph using the alternate approach. For the next academic year, he said he would teach the alternate approach first and then the traditional approach. He understood that teaching students to problem solve for one lesson wasn’t enough and would like to teach in this manner for the whole academic year.

4. As a result of faculty reflection on these results, are there any program changes anticipated?
   a. If so, what are those changes?

For Fall ’11, one lecturer and I will work on restructuring LS 7A, our introductory algebra class. I will teach a Monday-Wednesday-Friday class and the staff member will teach the Tuesday-Thursday class. This will allow us time to meet and visit each other classes. We will use common lesson plans and assessments.

   b. How will you know if these changes achieved the desired results?
I will document the pedagogical changes when I visit the lecturer’s classroom. Another marker for change will be the type of questions used in the tests and final exam. It is my hope that this staff member and I will begin to have discussions and meetings with other staff members on what we have learned from this.

5. **What assessment activities are planned for the upcoming academic year?**

For the upcoming year, the staff member and I will be embedding mathematical tasks into our assessments that will move students toward problem solving that relies on conceptual understanding rather than simply the application of procedural knowledge.
Bibliography


