Assessment Report  
Department of Mathematics and Statistics  
California State University Sacramento  
June 2011

Introduction.

The BA in Mathematics at CSUS requires that students complete a standard 18-unit lower division course sequence of mathematics courses as well as an introductory course in computer science. These courses include calculus, differential equations and linear algebra, and prepare students for the analytic rigor that underscores upper division mathematics and statistics courses as well as the fundamental tools to apply mathematics in lower division science and engineering. Mathematics majors are strongly urged to take elementary physics with calculus to better appreciate the importance of calculus based mathematical models throughout the sciences. All majors complete 15 units of upper division core study. This core consists of an Introduction to Formal Mathematics (Math 108) as well as a pair of yearlong sequences in Modern Algebra (Math 110A/B) and Functions of a Real Variable (Math 130A/B), classical subjects that form the foundation of all modern mathematics study. Besides these core topics, students are required to take an additional 12-15 elective units from their chosen specialty, emphasizing pure mathematics, applied mathematics, or preparation for secondary teaching. The pure mathematics option contains those subjects most likely to be of assistance in a postgraduate program, while the applied option stresses material in statistics and areas that have applications in industry as well as prepare the student for further study in the applications of mathematics. The secondary teacher preparation option is an approved waiver program for the secondary credential and has a specific set of electives separately approved by the state as a waiver for the state’s content examinations in mathematics.

The BA program provides all mathematics graduates with a common background in the core areas of mathematics: modern algebra and real analysis. With the depth and breadth of training of the major options, students are able to enter graduate programs or use their skills in the classroom or in the corporate and industrial world.

1. What goals or learning objectives/outcomes were assessed in the academic year ending June 30, 2011?

During the 2010-11 academic year, the Core Curriculum Committee in the department carried out one of the recommendations of the Academic Program Review Report to reword the first three departmental learning goals in order to make them clearer to a general readership. The Committee thus revised the previous learning goals and identified the following five learning goals for mathematics majors:

1. The mathematics major at CSUS is expected to become proficient at mathematical reasoning, and to understand the central role that logical rigor plays in all mathematics.
2. The mathematics major at CSUS is expected to develop a fundamental understanding of the main areas of mathematics: geometry, algebra, and analysis.

3. The mathematics major at CSUS is expected to develop an appreciation of the variety of areas of mathematics study and of mathematical applications.

4. The mathematics major at CSUS is expected to demonstrate an ability to effectively communicate mathematical thought.

5. The mathematics major at CSUS is expected to have a basic understanding of current technology in mathematics and to demonstrate skills with that technology.

The department felt that these learning goals at the department level connected well with the university's learning goals as well as national and international standards and trends. These goals lead to competence in the disciple of mathematics, inquiry and analysis, critical and creative thinking, and problem solving, as well as effective communication. Students also gain the foundations and skills needed for lifelong learning. These are, generally speaking, some of the learning objectives at most universities and are included in the learning goals at Sacramento State. The first four learning goals were assessed in the 2010-11 academic year.

In addition to rewriting the learning goals for the math major, the Core Curriculum Committee also worked on identifying learning goals specific to the two year long sequences that are required of all math majors, namely Math 110A/B (Modern Algebra) and Math 130A/B (Functions of a Real Variable). In addition to specific learning goals, the Committee also included a preamble to the learning goals on the importance of effective communication of mathematical ideas in written form. The text that follows is the Committee's recommendation that the Department adopt these principles of writing in the core courses and the learning goals for Math 110 and 130. The department approved these principles and goals at its meeting on May 5, 2011.

The Department of Mathematics and Statistics has as a goal in all of its Core Curriculum classes (Math 108, 110A/B, 130A/B) that students be able to effectively communicate mathematical ideas in written form. This could include clear written explanations of mathematical ideas as well as constructed formal proofs. The writing allows students to reflect on their learning and deepen their understanding of the concepts in the courses. It is a useful aspect for understanding the language of mathematics and allows students to express themselves clearly in this language.

Math 110A/B Learning Goals

Math 110A students will be able to:

1. identify whether a specific set along with an operation form a group, and whether a specific subset forms a subgroup.
2. apply basic theorems about orders of elements, groups, and subgroups to specific examples.

3. prove results about orders of elements and generators of cyclic groups.

4. work fluently with examples of permutations groups, using cycle notation, representations of permutations as products of disjoint cycles and as products of transpositions.

5. identify whether a mapping of groups is a homomorphism.

6. use the kernel of a homomorphism to identify whether a homomorphism is a monomorphism.

7. use the correspondence theorem to identify subgroups and normal subgroups of homomorphic images of groups.

8. work fluently with coset arithmetic in quotient groups.

Math 110B students will be able to:

1. identify whether a set with multiplicative and additive operations is a ring.

2. identify whether a subset of a ring is an ideal.

3. understand basic relationship between a commutative ring and the ring of polynomials with coefficients in that ring.

4. prove basic results about homomorphisms of rings, and use the kernel to identify whether a homomorphism is a monomorphism.

5. work fluently with coset arithmetic in factor rings.

6. use the correspondence theorem to identify ideals of homomorphic images of rings.

7. recognize rings of polynomials as a generalization of the integers, with factorization properties depending on the properties of the ring of coefficients.

8. identify degrees of field extensions.
Math 130A/B Learning Goals

Math 130A/B students will be able to:

1. demonstrate an understanding of the formal epsilon-delta definitions of limits, continuity, differentiability and integrability and be able to establish basic results using these definitions.

2. prove the fundamental theorems, including the Intermediate Value Theorem, Rolle's Theorem, the Mean Value Theorem, l'Hopital's Rule, and the Fundamental Theorem of Calculus.

3. apply the results of the major theorems to both standard and non-standard exercises.

4. demonstrate an understanding of the properties of the real numbers, such as finding an infimum and supremum of specific sets, and using the Archimedean property.

5. distinguish between continuity and uniform continuity and understand the connection between differentiability and continuity.

6. identify whether a function is integrable and show that continuous functions on a closed interval are integrable.

7. determine whether a sequence satisfies the Cauchy condition and prove whether a sequence converges or diverges.

8. understand the difference between pointwise convergence and uniform convergence for a sequence of real valued functions.

2. How did you assess these learning outcomes?

Final exams from the courses Math 110A, Math 110B, (Modern Algebra) and Math 130A, Math 130B (Functions of a Real Variable) are reviewed by the department chair and the chair of the core curriculum committee to ensure that the learning goals of the major are being met by the students who pass the exams and the courses. Each of these courses employs a two-hour comprehensive final exam. The instructors submit three exams from each course, one representing exemplary work, one representing superior work, and one representing satisfactory work (provided all three examples exist). The students’ names are removed from the exams so that the work is anonymous when reviewed. The University’s Faculty Assessment Coordinator has deemed this exam review process that is in place as solid. The review is designed to determine whether there are a sufficient number of questions addressing each learning goal to ensure that a student who answers most of the questions correctly will have met the learning goal. An individual question may address several learning goals. The exams were also reviewed to
make sure that they accurately reflected the required course content, and to see how similar items appear on the exams.

Suitable final exams should contain questions requiring detailed exposition of reasoning (goal 1). The strands of algebra and analysis should appear regularly in the finals of Math 110 and Math 130, and to a lesser extent, we anticipate questions concerning the strand of geometry (goal 2). Other areas of study, such as number theory, set theory, relations, functions, history of mathematics, and applications may appear as well (goal 3). All the finals should contain questions requiring independent thought and clearly written explanations (goal 4). Although there are ways in which students may employ the appropriate use of technology in the core courses, the conceptual core of mathematics is not based on the use of technology and the core courses may, but are not expected to, address goal 5. Other courses in the curriculum, such as calculus, differential equations, and the computer science course that all math majors are required to take do employ technology as a part of their curriculum and goal 5 is met through these courses.

Final exams from Math 110A/B and Math 130A/B were examined to see whether or not students who passed these exams were meeting the learning goals for the math major. For Math 110A, 38 of the 51 questions did a good job assessing learning goal 1 dealing with proficiency in mathematical reasoning and the central role that logical rigor plays in mathematics. Learning goal 2 on developing an understanding of the main strands of mathematics was adequately addressed by 46 of the 51 questions, while 13 of the questions addressed goal 3 on understanding the breadth of mathematics. Learning goal 4 on effective communication of mathematical thought was assessed by 39 of the questions. For Math 110B, where there were a total of 39 questions on the final exams, all 39 addressed learning goals 1 and 2, while 18 addressed learning goal 3, and 33 addressed learning goal 4. In Math 130A and Math 130B, virtually all the 48 questions did a good job of assessing goals 1, 2, and 4. Learning goal 3 was addressed by 8 of the 48 questions. Overall the exams did a fine job of examining the learning goals of the major. The student responses on the exams ran the usual gamut from highly competent to marginally capable, but overall the general feel is that students do have a good sense of what is expected of a math major. The students are capable of putting together a competent proof using a definition. While students seem more comfortable in applying some of the more important results, they clearly see the importance of knowing the major theorems and for the most part are able to offer a capable attempt at the foundations of Modern Algebra and Real Analysis.

Turning next to the specific goals of Math 110A/B and Math 130A/B, the exams and student responses were evaluated with these specific goals in mind. Recognizing that these goals were not established by the department until May of this year, instructors of the two year-long sequences did not have these specific goals in mind as they taught theses classes over the academic year. Nonetheless, the final exams addressed the specific goals well, and student responses indicate that they are meeting the specific goals of the courses. In the Math 110A/B sequence, only learning goal 4 concerning working with permutation groups lacked adequate coverage in the final exams. Although there
was one question dealing with permutation groups, there were no questions using cyclic notation, representations of permutations as products of disjoint cycles or transpositions. The lack such a question is not of great concern, since it is almost certain that there was coverage of such material during the course on midterm exams or homework. The fact that there was one question on permutation groups does indicate that the material was covered. Also, in other questions asking for examples, some students used permutation groups to illustrate a point, and in their responses, they used cyclic notation and did write permutations as products of disjoint cycles. In the Math 130A/B sequence, all the exams included a number of questions that required students to develop a proof using epsilon-delta definitions of limits, continuity, differentiability and integrability. Students showed a range of abilities in tackling these problems, and for the most part demonstrated a basic comprehension of the methods employed. One trend that is most evident from the exams is that Math 130A lends itself to a much wider range of epsilon-delta problems than does Math 130B. In Math 130A questions concerning the infimum and supremum of a set as well as questions about limits and continuity provide a wealth of possibilities for examining the use of epsilon-delta methods. Since Math 130B uses many of the results from Math 130A, the range of definition type problems is somewhat more limited and exam questions were more likely to focus on the application of the theory that had been developed in class. All the exams also included questions on some of the major results listed in learning goal 2 of Math 130A/B. Students were able to give reasonable proofs of this material. All the exams also asked questions that required the application of some of the important theorems. Students were able to see the connections between the appropriate theorem and the application and were able to establish some important and interesting results. Learning goals 4 - 8 related to various topics that were studies in Math 130A, and while not all of these topics showed up on final exams, a number of the important concepts were examined. Two of the exams included questions about supremaums and infimums, another exam asked for a proof of the Archimedean property, one exam examined the integrability of a continuous function and another had a wonderful question on uniform continuity. Many of the topics listed in the learning goals were touched upon and the students demonstrated a sound comprehension of the material.

Pass rates in the courses were recorded to measure the relative success at achieving the learning goals. Pass rates between 65 and 75 percent in the first course of each sequence indicate successful achievement of learning outcomes. Pass rates between 85 and 100 percent in the second course of a sequence indicate successful achievement of learning outcomes. The higher pass rate in the second course is also an indication that the students who passed the first course are in fact achieving the learning goals and are well prepared for the second course.

For the 2010-11 academic year, the pass rates were as follows:

<table>
<thead>
<tr>
<th>Course</th>
<th>Number of Sections</th>
<th>Pass rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 110A</td>
<td>3</td>
<td>63.0%</td>
</tr>
<tr>
<td>Math 110B</td>
<td>2</td>
<td>79.4%</td>
</tr>
<tr>
<td>Math 130A</td>
<td>2</td>
<td>50.0%</td>
</tr>
<tr>
<td>Math 130B</td>
<td>3</td>
<td>90.0%</td>
</tr>
</tbody>
</table>
Pass rates in three of the four courses are below the desired range. However, of the students enrolled and receiving non W grades in Math 110A and Math 110B, if an additional two students had passed in each class, the pass rate would have been in the desired range for those classes, so it is felt that no major changes need to be undertaken in Math 110A/B. Of greater concern is the pass rate in Math 130A at 50%. This is below the desired rate. The annual pass rates in Math 130A were checked going back five years to see if any trends could be determined. The following table shows these results:

<table>
<thead>
<tr>
<th>Year</th>
<th>2005-06</th>
<th>2006-07</th>
<th>2007-08</th>
<th>2008-09</th>
<th>2009-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass rate in Math 130A</td>
<td>84.6%</td>
<td>70.2%</td>
<td>69.2%</td>
<td>73.1%</td>
<td>63.4%</td>
</tr>
</tbody>
</table>

This evidence suggests that the 50% pass rate in 2010-11 was an outlier, but attention should be maintained on this statistic in future years.

Complete grade distribution of each core course in 2010-11 appears in the following table:

2010-11 Math 110A/B, Math 130A/B grade distribution

<table>
<thead>
<tr>
<th>Grade</th>
<th>Math 110A</th>
<th>Math 110B</th>
<th>Math 130A</th>
<th>Math 130B</th>
<th>Total</th>
<th>% Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>17</td>
<td>8.2</td>
</tr>
<tr>
<td>A-</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>7</td>
<td>11</td>
<td>5.3</td>
</tr>
<tr>
<td>B+</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>13</td>
<td>6.3</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>18</td>
<td>8.7</td>
</tr>
<tr>
<td>B-</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>3.9</td>
</tr>
<tr>
<td>C+</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>10</td>
<td>4.8</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>6</td>
<td>3</td>
<td>8</td>
<td>37</td>
<td>17.9</td>
</tr>
<tr>
<td>C-</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>27</td>
<td>13.0</td>
</tr>
<tr>
<td>D+</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>22</td>
<td>10.6</td>
</tr>
<tr>
<td>D-</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>1.9</td>
</tr>
<tr>
<td>F</td>
<td>14</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>17</td>
<td>8.2</td>
</tr>
<tr>
<td>WU</td>
<td>7</td>
<td>0</td>
<td>12</td>
<td>3</td>
<td>22</td>
<td>10.6</td>
</tr>
<tr>
<td>Total</td>
<td>83</td>
<td>34</td>
<td>44</td>
<td>50</td>
<td>207</td>
<td></td>
</tr>
</tbody>
</table>

In addition to evaluating the final exams in Math 110A/B and Math 130A/B, another method of assessment is a report submitted by the instructor of Math 193 (The Capstone Course for the Teaching Credential Candidate). This year’s report was filed following completion of Math 193 at the end of the spring semester 2011. The Capstone Course has several prerequisites, and as such, offers an opportunity to assess students who are close to the end of their undergraduate academic careers. Most students taking the course are within one semester of graduation. Students enrolled in Math 193 each give several presentations to the class (goal 4). These talks draw connections between the mathematics the students have learned in college and the secondary mathematics
curriculum. This course provides a unique perspective on the students, because in addition to seeing the students in their traditional roles, the instructor is able to see them as presenters of mathematics.

This year's report began by commenting on the quality of the lower division mathematical background of the students. In general, their grasp of lower division mathematics was solid. Many of the students had tutored lower division mathematics for years, so this was not surprising. Their understanding of the topics, especially precalculus material, was overly formulaic. They could solve the routine sorts of problems, but their understanding of the mathematics behind the solution was sometimes not solid. When they were shown a glimpse of the mathematical explanation, they moved very quickly to the fuller understanding that is desired. Geometry was a particular area of strength for these students. Students were generally familiar with the traditional proofs and explanations for the lower division mathematics that was encountered. They were not generally familiar with more hands-on or visual ways of explaining and proving that mathematics. This is part of the point of the Capstone Course, to provide future teachers with more of these ways of approaching traditional mathematical topics.

Next the report discussed the understanding of the core subjects, as well as the breadth of understanding that the students possessed. The students did an extended exercise that required significant applications of group theory (Math 110A), and as a class, they were strong in this area. Use was made of field theory (Math 110B), and there were several students who were fluent in this area, although many students had not yet taken the second semester of the modern algebra course, so they were not familiar with fields. The other portion of the core, real analysis (Math 130A/B), was used less significantly in the Capstone Course. Students did make use of convergence of sequences, although mostly at a calculus level, but the class' understanding was fine. Most of the students in the class had completed the courses that they take in order to achieve the breadth of understanding that we expect of our future teachers, i.e., number theory, geometry, and history of mathematics. In the Capstone Course the instructor made extensive use of their knowledge of these topics. The students were very familiar with basic number theory and enthusiastic about seeing applications of number theory. Many students chose their presentation topics accordingly. Likewise, geometry was a popular topic for presentation. Especially in these two areas, the instructor was impressed by the willingness of the students to dive into problems. They certainly are comfortable with the prospect of working their way through a geometry problem that requires a sequence of steps. The students also used basic computer technology in their presentations, such as MS PowerPoint, but the main use of technology was graphing calculators. They are clearly comfortable with this technology. (Goal 5)

Finally the report commented on the skills in mathematical communication of the students. The quality of the presentations given by the students in the course varied greatly. The students were not always comfortable in this role, and they were not always smooth in their deliveries, but they were consistently enthusiastic and interested in improving. Over the course of the semester, in which each student made three presentations, there was significant improvement for every student. The students were
apparently used to speaking with other students about mathematics. It appears that as they are asked to work extremely hard in their core classes to solve very challenging problems, they have learned that discussing mathematics with each other is a crucial strategy. In this way, they have learned to use mathematical vocabulary in talking with other students. While this might not be a part of the department's plan in its core courses, it certainly is a desirable outcome. Each student was asked to provide a written lesson plan for each of their presentations as well as a written self-evaluation of each presentation after it was over. Their writing skills were good. One last aspect of their communication of mathematics that was most striking was their inventiveness. They put a great deal of thought into their presentations and developed some very clever ways of looking at secondary mathematics. Sometimes this was a great example that they concocted and sometimes it was a manipulative that they built for a specific presentation. It was impossible not to be impressed by the creativity they showed in their presentations. As a final comment, the instructor was especially impressed by the enthusiasm of the students for learning mathematics.

3. **As a result of faculty reflection on these results, are there any program changes anticipated?**

No major changes will be implemented as a result of this year's assessment activities. The most recent Department Review was sent to the senate in spring of 2011 for its final approval. One aspect of that report was the following commendation: By maintaining the 15-unit core for all three concentrations of the B.A., the Department has fostered integrity in the undergraduate program. The department feels that its assessment activities this year has reinforced the belief that the strength of its program is maintained by the strength of its core. In the Report of the External Consultant, it was noted that "the Department is in good position to defend its assessment plan in future reviews precisely because the core of its major is so strong. It is unusual across both university systems in California for an undergraduate program degree in mathematics to require a full year each of modern algebra and analysis (typically the second semester of these subjects are electives), and even more unusual for this requirement to apply to all strands within the major."

4. **Did your department engage in any other assessment activities such as the development of rubrics, course alignment?**

In comparing this year's data with last year, one question that arose was whether students who were unsuccessful in Math 110B or Math 130B went on to ultimately successfully complete the course that they did not pass when the original data was collected. In the 2009-10 academic year there were eight different students who attempted Math 110B or Math 130B that did not pass. (One of these did not pass Math 110B in both Fall 2009 and Spring 2010.) Of these eight students, five passed the course in the next semester, and one passed on the third attempt at the course. The remaining two of the eight attempted the course one more time (this included the student who did not pass Math 110B in both Fall 2009 and Spring 2010) but were unsuccessful in that subsequent attempt as well. Neither of these attempted the course again and neither attended CSUS in Spring 2010.
The department also contacted the 36 students graduating with Math degrees during the fall or spring in 2010-11 in order to survey them as to their plans for the coming year. Of these 36 students, 20 completed the survey. Of these 20, seven will be attending a teaching credential program in Fall 2011 and two more plan to apply for entry into a credential program within the next year. Eight plan to attend graduate school in mathematics or statistics in Fall 2011. Three are seeking employment. Since ultimately the success of a program is measured by whether the students going through the program are successful in the subsequent stage of their life and career, these results show that the mathematics program is successful.

5. **What assessment activities are planned for the upcoming academic year?**

Over the past year, as part of revising the five leaning goals, it was felt that the math faculty came to know and understand the learning goals well. One of the goals of the coming year will be to make sure that faculty link their class learning outcomes to the department learning outcomes. With this in mind, there are planned discussions of how this can be accomplished. One of the main strengths of the upper division core courses is the writing component of the courses. Efforts will be made to ensure some uniformity in the writing components of the upper division core.