Learning Outcomes Data for the Senate Committee on Instructional Program Priorities

Template
Program: Mathematics MA
Department: Mathematics and Statistics
Number of students enrolled in the program in Fall, 2011: 28
Faculty member completing template: Tracy Hamilton Date: February 3, 2012

Period of reference in the template: 2006-07 to present
1. Please describe your program’s learning-outcomes trajectory since 2006-07: Has there been a transformation of organizational culture regarding the establishment of learning outcomes and the capacity to assess progress toward their achievement? If so, during which academic year would you say the transformation became noticeable? What lies ahead; what is the next likely step in developing a learning-outcomes organizational culture within the program?

[Please limit your response to 200 words or less]

The department recently developed its learning objectives for our MA program.

A few years ago the department developed learning objectives and assessment plans for the departments main program, its BA in Mathematics. Since that time, attention has been focused on making that assessment plan work well. The fundamental issue has been the relationship between the learning objectives for the programs and the learning objectives for the core courses in the programs, and the balance between depth and breadth and how that should be assessed. It seems to be the nature of our discipline that the core courses determine the depth and quality of the program. Hence the recent focus on learning objectives for core courses.

With the knowledge and experience that we have from that process, the department has moved forward with learning objectives for its other degree programs.

2. Please list in prioritized order (or indicate no prioritization regarding) up to four desired learning outcomes (“takeaways” concerning such elements of curriculum as perspectives, specific content knowledge, skill sets, confidence levels) for students completing the program. For each stated outcome, please provide the reason that it was designated as desired by the faculty associated with the program.

[Please limit your response per outcome to 300 words or less]

No prioritization:

a) A recipient of an MA in mathematics from CSUS is expected to have a deep understanding of the fundamental theorems and techniques in both abstract algebra and real analysis. This includes the development of these disciplines from first principles.
It is generally recognized that advanced study in mathematics requires a solid background in the areas of real analysis and modern algebra. These two strands represent a classical approach to the subject that is still essential learning for any modern study of the subject.

This classical approach distinguishes the CSUS MA in Mathematics from many others. Most MA programs at state universities emphasize breadth at the expense of depth of knowledge. Most mathematics MA programs at doctorate-granting institutions fail to set clear expectations for depth of understanding. We believe our program to be of the very highest quality. This belief was enthusiastically expressed by the outside reviewer in our most recent program review.

Students are expected to complete a full year of study of both real analysis and modern algebra, and are expected to demonstrate the ability to prove and explain some of the fundamental results from these areas.

b) A recipient of an MA in mathematics from CSUS is expected to have a mathematical sophistication that allows them to apply their understanding to problems that they have not seen before and in contexts that they have not seen before. The ability to be creative with the application of basic knowledge is a hallmark of a sophisticated mathematical thinker.

The department believes that the masters degree in mathematics indicates a mastery of the creative process in mathematics, which is the original thought required to apply knowledge of fundamental results to new problems and situations. This distinguishes the MA from the BA in mathematics.

c) A recipient of an MA in mathematics from CSUS is expected to speak the language of mathematics fluently, to reason with impeccable mathematical rigor, and to do this by designing proofs of mathematical results.

Professional mathematicians regard mathematical proof as the intrinsic essence of mathematics, and it is expected that our graduate students will arrive at an appreciation for the role of proof in mathematical discourse, as well as a grasp of the methods of proof that permeate all mathematical exposition.

The ability to communicate mathematical ideas should be expected of all students graduating with a degree in mathematics. This ability goes to the heart of the mathematical process and centers on the need for clear logical presentation and exposition.

Students should be able to identify various methods of proof, and apply these methods to their work in their courses. Application of these fundamental mathematical methods leads to a deeper insight into the nature of the subject.
d) A recipient of an MA in mathematics from CSUS is expected to have an appreciation of the variety of major modern areas of mathematics study and of mathematical applications.

The study of mathematics has been an integral part of mankind’s intellectual history for over two thousand years, and in many ways approaches the pinnacle of mankind’s intellectual accomplishments. During the past two thousand years, the nature of mathematical inquiry has expanded to include a wide range of topics, from the classical studies of geometry and number theory to include modern subjects of interest such as topology, chaos theory and game theory. Current mathematical studies range over a wide variety of courses and often include interdisciplinary exchanges.

3. *For undergraduate programs only,* in what ways are the set of desired learning outcomes described above aligned with the University’s Baccalaureate Learning Goals? Please be as specific as possible.

[Please limit your response to 400 words or less]

Not applicable to this graduate program.

4. For each desired outcome indicated in item 2 above, please:
   a) Describe the method(s) by which its ongoing pursuit is monitored and measured.
   b) Include a description of the sample of students (e.g., random sample of transfer students declaring the major; graduating seniors) from whom data were/will be collected and the frequency and schedule with which the data in question were/will be collected.
   c) Describe and append a sample (or samples) of the “instrument” (e.g., survey or test), “artifact” (e.g., writing sample and evaluative protocol, performance review sheet), or other device used to assess the status of the learning outcomes desired by the program.
   d) Explain how the program faculty analyzed and evaluated (will analyze and evaluate) the data to reach conclusions about each desired student learning outcome.

[Please limit your response to 200 words or less per learning outcome]

*(If the requested data and/or analysis are not yet available for any of the learning outcomes, please explain why and describe the plan by which these will occur. Please limit your response to 500 words or less.)*

Learning outcome: A recipient of an MA in mathematics from CSUS is expected to have a deep understanding of the fundamental theorems and techniques in both abstract algebra and real analysis. This includes the development of these disciplines from first principles.

a) Students in the MA program take Math 210A/B (Algebraic Structures) and Math230A/B (Real Analysis). Each of these is a yearlong sequence of two courses. The culminating experience for the Master’s degree consists of a 3 hour comprehensive exam in abstract algebra and a 3 hour
comprehensive exam in real analysis. Questions for the exams are submitted by professors who have recently taught the sequences, and the questions are reviewed by the graduate coordinator. Each completed exam is read by two professors, and if there is any discrepancy in their judgment of the results, the graduate coordinator acts as an arbitrator.

b) The comprehensive exams are taken by all students obtaining the MA in Mathematics. The exams are given every semester if needed.

c) The recent comprehensive exam questions are attached as Appendix A.

d) As noted previously, two professors review the completed exams. They look for an understanding of the theorems and techniques in the solutions that the students have written.

Learning outcome: A recipient of an MA in mathematics from CSUS is expected to have a mathematical sophistication that allows them to apply their understanding to problems that they have not seen before and in contexts that they have not seen before. The ability to be creative with the application of basic knowledge is a hallmark of a sophisticated mathematical thinker.

a) Some questions on the comprehensive exams will be problems that the students have not seen before and in contexts that they have not seen before. Their solutions to these problems will indicate the degree to which this learning outcome is met.

b) The comprehensive exams are taken by all students obtaining the MA in Mathematics. The exams are given every semester if needed.

c) The recent comprehensive exam questions are attached as Appendix A.

d) Two professors review the completed exams. They look for the ability of the student to apply their knowledge of algebra and analysis to these problems.

Learning outcome: A recipient of an MA in mathematics from CSUS is expected to speak the language of mathematics fluently, to reason with impeccable mathematical rigor, and to do this by designing proofs of mathematical results.

a) Questions on the comprehensive exams are ones that require mathematical rigor and proofs. The graduate coordinator acts as the overseer to make sure that this is the case when putting together the exams.

b) The comprehensive exams are taken by all students obtaining the MA in Mathematics. The exams are given every semester if needed.
c) The recent comprehensive exam questions are attached as Appendix A.

d) Two professors review the completed exams. They look for the students to show mathematical rigor and clearly developed proofs.

Learning outcome: A recipient of an MA in mathematics from CSUS is expected to have an appreciation of the variety of major modern areas of mathematics study and of mathematical applications.

a) In addition to the two yearlong sequences in algebra and analysis that student in the Master's program take, they also complete additional courses in Topology (Math 220A/B), Complex Analysis (Math 234A/B), Methods of Applied Mathematics (Math 241A/B), or Mathematical Statistics (Stat 215A/B). Because this learning outcome is a breadth outcome, it is not as amenable to assessment by exams as the other outcomes. Instead, it is the variety of other classes that is in place that determines the genuine breadth of the student experience.

b) All students in the Master's program take at least some of the courses listed in part (a) above. The graduate coordinator acts as the advisor to students in the program and has control over the breadth requirement when designing the student's coursework. Beginning in 2011-12, the department began offering single semester courses rather than yearlong sequences in the breadth areas of Topology, Complex Analysis, Applied Math, and Statistics. This allows students to see more of a variety of topics.

c) N/A

d) There is a Graduate Committee from the department, chaired by the graduate coordinator, and generally made up of faculty that have recently taught the graduate level courses. This committee meets to discuss specific requirements for the MA program. The appropriates of the breadth requirement is under frequent consideration by this committee, and this led to the changing of traditional yearlong sequences to single semester offerings.

5. Regarding each outcome and method discussed in items 2 and 4 above, please provide examples of how findings from the learning outcomes process have been utilized to address decisions to revise or maintain elements of the curriculum (including decisions to alter the program’s desired outcomes). If such decision-making has not yet occurred, please describe the plan by which it will occur.

[Please limit your response to 200 words or less per item]

a-c) Our first three learning outcomes are all monitored by looking at the comprehensive exam (culminating experience). Overall, we have felt that we are meeting these outcomes. However, there was some concern by the graduate committee that the real analysis portion of this exam was not meeting our needs. In particular, we did not feel that it met learning outcome (b)
because too many of the questions were standard problems that the student would have seen before. Through conversation within the graduate committee, the faculty involved in writing the real analysis portion of the exam have begun to transform what they are doing in the real analysis course and those changes are reflected in the real analysis comprehensive exam.

d) Over the last few years, we have heard from students that they would like to see more variety in our graduate program. Also, the graduate committee felt that the program may be lacking some breadth. However, due to budget constraints, we are limited to offering one 200-level elective per semester. This led to the decision to offer four one-semester electives during each two-year period instead of offering two two-semester electives during each two-year period. This change also allowed us to offer a statistics course as one of the electives. We felt that this would help our students when looking for teaching jobs since it would likely make them eligible to teach statistics courses as well as mathematics courses.

6. Has the program systematically sought data from alumni to measure the longer-term effects of accomplishment of the program’s learning outcomes? If so, please describe the approach to this information-gathering and the ways in which the information will be applied to the program’s curriculum. If such activity has not yet occurred, please describe the plan by which it will occur.

[Please limit your response to 300 words or less]

We are currently in the process of searching for alumni who have completed our program in the past 10 years. Of the 43 students in this category, we have located 36 of them so far.

Our program is designed to serve two categories of students: those who plan to teach at a community college and those who plan to go on to a Ph.D. program. Of the 36 recent graduate that we were able to locate, we found that 28 (78%) of them are currently teaching at a community college or here at CSUS and 7 (19%) of them are either currently pursuing a Ph.D. or have completed a Ph.D. program. These findings support our feeling that the program we have now is serving our students well.

7. Does the program pursue learning outcomes identified by an accrediting or other professional discipline-related organization as important? Does the set of outcomes pursued by your program exceed those identified as important by your accrediting or other professional discipline-related organization?

[Please limit your response to 300 words or less]

The program does not pursue learning outcomes identified by an accrediting or other professional discipline related organization.

8. Finally, what additional information would you like to share with the Senate Committee on Instructional Program Priorities regarding the program’s desired learning outcomes and assessment of their accomplishment?
Appendix A

[Please limit your response to 200 words or less]
COMPREHENSIVE EXAM
ALGEBRA
May 2011

Part I: Group Theory (Do 4 of the following 5 problems)

1. Let $G$ be a group

   (a) Prove that if $G/Z(G)$ is cyclic, then $G$ is Abelian. (Note: $Z(G)$ denotes the center of $G$.)
   
   (b) Prove that $G/Z(G)$ is isomorphic to $\text{Inn}(G)$. (Note: $\text{Inn}(G)$ denotes the group of inner automorphisms on $G$.)
   
   (c) Prove that if $\text{Inn}(G)$ has more than one element, then $\text{Inn}(G)$ is not cyclic.

2. 

   (a) Describe all conjugacy classes in $S_4$ and determine the size of each class.
   
   (b) Let $\sigma = (1\ 2) \in S_4$. Determine, with explanation, the centralizer of $\sigma$ in $S_4$.
   
   (c) Let $n \geq 3$. Let $H \leq S_n$ with $|H| = 2$. Prove that $H$ is NOT normal in $S_n$.
   
   (d) Prove that if $n \geq 3$, then there does not exist a homomorphism from $S_n$ onto $A_n$.

3. Let $G$ be a group such that $|G| = 385 = 5 \cdot 7 \cdot 11$.

   (a) Give, with explanation, all possibilities for the number of subgroups of $G$ of order 5.
   
   (b) Suppose $G$ has a normal subgroup of order 5. Prove that $G$ is cyclic.

4. Let $G$ be an Abelian group of order 120.

   (a) Determine, up to isomorphism, all Abelian groups of order 120.
   
   (b) For each isomorphism class found in part (a), how many elements are there of order 5? how many subgroups are there of order 5?
   
   (c) Suppose $G$ has three elements of order 2, determine the isomorphism class of $G$.

5. 

   (a) State Cauchy’s Theorem.
   
   (b) Assume Cauchy’s Theorem holds for Abelian groups. Prove Cauchy’s Theorem holds for non-Abelian groups.
Part II: Ring and Field Theory (Do 4 of the following 5 problems)

1. Let \( R \) be the ring of continuous functions on the closed interval \([0, 1]\). Let \( M = \{ f \in R : f \left( \frac{1}{2} \right) = 0 \} \).
   
   (a) Prove that \( M \) is an ideal in \( R \).
   
   (b) Prove that \( R/M \) is isomorphic to \( \mathbb{R} \).
   
   (c) Prove that \( M \) is a maximal ideal in \( R \).

2. Let \( F \) be a field.
   
   (a) Suppose \( E \) is an algebraic extension of \( F \). Prove that every subring of \( E \) containing \( F \) is a field.
   
   (b) Give a counterexample to show that the result in part (a) is not necessarily true if \( E \) is NOT an algebraic extension of \( F \).

3. Let \( S \) be a Euclidean domain.
   
   (a) Prove that \( S \) is a ring with unity.
   
   (b) Prove that \( S \) is a principal ideal domain.
   
   (c) Let \( I = (a) \) be an ideal in a principal ideal domain. Prove that \( I \) is a maximal ideal if and only if \( a \) is irreducible.

4. Let \( K = \mathbb{Q} \left( \sqrt{2}, i \right) \) and let \( F_1 = \mathbb{Q} (i) \) and let \( F_2 = \mathbb{Q} \left( \sqrt{2} \right) \). Prove each of the following
   
   (a) \( \text{Gal}(K/F_1) \cong \mathbb{Z}_8 \)
   
   (b) \( \text{Gal}(K/F_2) \cong D_4 \) (where \( D_4 \) denotes the dihedral group with 8 elements).

5. Determine each of the following (with explanation).
   
   (a) All ideals in \( \mathbb{Q}[x] / (x^4 + 4x - 5) \).
   
   (b) All fields which are homomorphic images of \( \mathbb{Z}_{60} \).
   
   (c) Describe all subfields of \( \mathbb{C} \) which are homomorphic images of \( \mathbb{Q}[x] / (x^3 + 2x) \).
Master's Exam in Real Analysis  
May 2011


1. Let \( a = (a_1, ..., a_n), b = (b_1, ..., b_n) \in \mathbb{R}^n \).
   (a) Show that
   \[
   \left( \sum_{i=1}^{n} a_i b_i \right)^2 \leq \sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2 .
   \]
   (b) Show that
   \[
   ||a|| = \left( \sum_{i=1}^{n} a_i^2 \right)^{\frac{1}{2}}
   \]
defines a norm on \( \mathbb{R}^n \).

2. 
   (a) Describe the construction of the Cantor set.
   (b) Show that the Cantor set is a perfect set.

3. Let \( X \) be a metric space and \( K \) be a compact subset of \( X \). Show that if \( F \) is closed and \( F \subset K \), then \( F \) is compact.

4. Let \( x_1 = 3 \) and
   \[
   x_{n+1} = \frac{1}{2} \left( x_n + \frac{2}{x_n} \right) , \quad \forall \ n \in \mathbb{N} .
   \]
   Show that \( \{x_n\} \) is a convergent sequence and find its limit.

5. Let \( \{x_n\} \) be a real sequence converging to \( x \in \mathbb{R} \). Show that
   \[
   a_n = \frac{x_1 + ... + x_n}{n} \rightarrow x .
   \]

6. Determine the interval of convergence for the following real power series
   \[
   \sum_{n=1}^{\infty} \frac{n^2}{3^n} x^n .
   \]
   Hint: Check the endpoints, too.

7. Let \( f : [0, 1] \rightarrow \mathbb{R} \) be defined by
   \[
   f(x) = \begin{cases} 
   \frac{1}{n} & \text{if } x = \frac{m}{n} \text{ in lowest terms} \\
   0 & \text{if } x = 0 \text{ or } x \notin \mathbb{Q} .
   \end{cases}
   \]
   Investigate the continuity of \( f \) on \([0, 1]\).

8. Let \( f \) be a continuous mapping of a compact metric space \( X \) into a metric space \( Y \). Prove that \( f \) is uniformly continuous.
Part 2. - Solve 6 problems from Part 2.

9. Let \( f : [0, 1] \to \mathbb{R} \) be differentiable on \((0, 1)\). Show that if for some \( L \geq 0 \), \(|f'(x)| \leq L\), for all \( x \in (0, 1) \), then \( \lim_{n \to \infty} f(2^{-n}) \) exists.

10. Let
\[
f(x) = \begin{cases} 
  x^3 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\
  0 & \text{if } x = 0.
\end{cases}
\]

Study the continuity and differentiability of \( f \) on \([0, 1]\).

11. List the rational numbers from the interval \([0, 1]\) as a sequence \( \{r_n\} \). Define \( f : [0, 1] \to \mathbb{R} \) by
\[
f(x) = \begin{cases} 
  \frac{n}{1+n^2} & \text{if } x = r_n \\
  0 & \text{if } x \notin \mathbb{Q} \cap [0, 1].
\end{cases}
\]

Use lower and upper Riemann sums to determine whether or not \( f \) is Riemann-integrable on \([0, 1]\).

12. Let
\[
\alpha(x) = \begin{cases} 
  0 & \text{if } x < 0 \\
  1 & \text{if } x = 0 \\
  2 & \text{if } x > 0.
\end{cases}
\]

Show that \( f \in \mathcal{R}(\alpha) \) on \([-1, 1]\) if and only if \( f \) is continuous at \( x = 0 \).

13. Consider \( f_n(x) = n 2^{-(n-1)x}, \ n \in \mathbb{N} \) and let \( 0 < a < 1 \).

(a) Show that the series \( \sum_{n=1}^{\infty} f_n \) converges uniformly on \([a, \infty)\).

(b) If \( f(x) = \sum_{n=1}^{\infty} f_n(x) \) for \( x \geq a \), find \( f'(1) \).

14. Let
\[
f_n(x) = \frac{nx}{1+n^2 x^2}, \ n \in \mathbb{N}.
\]

Investigate the pointwise and uniform convergences of the sequence \( \{f_n\} \) and of the series \( \sum_{n=1}^{\infty} f_n \) on \([0, 1]\).

15. Consider a sequence of functions \( \{f_n\} \) defined on \([a, b]\). Show that if each \( f_n \) is Riemann integrable on \([a, b]\) and the sequence \( \{f_n\} \) is uniformly bounded, then the sequence \( \{F_n\} \), defined by
\[
F_n(x) = \int_{a}^{x} f_n(t) \, dt
\]
contains a uniformly convergent subsequence on \([a, b]\).

16. Let \( \alpha \) be monotonically increasing and bounded on \([a, b]\). Suppose that \( f_n \in \mathcal{R}(\alpha) \) on \([a, b]\), for all \( n \in \mathbb{N} \) and that \( f_n \) converges uniformly to \( f \) on \([a, b]\).

Prove that \( f \in \mathcal{R}(\alpha) \) and
\[
\int_{a}^{b} f \, d\alpha = \lim_{n \to \infty} \int_{a}^{b} f_n \, d\alpha.
\]