A study of inference problems for non-linear functional data

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Project Objectives

Alzheimer's disease, an age-related neurodegenerative disorder, progresses from a healthy brain condition to a mild cognitive impairment and finally to irreversible neuronal loss (Hani et al., 2019). Functional magnetic resonance imaging (fMRI) is considered a promising tool to detect Alzheimer's disease. The fMRI data - data consisting of signals of fMRI scans - is one of the most popular data used in the literature to study fMRI's use for Alzheimer's disease detection. Most of the existing literature consider the fMRI data as "linear" in nature and use tools from Functional Data Analysis (FDA). However, several studies suggest that the fMRI data are "non-linear" in nature. When the data is non-linear, the tools from FDA are not directly applicable. Recently, several methods have been proposed to study different aspects of non-linear data (Petersen & Mueller, 2016). In this project, we propose two methodologies to study inference problems, such as construction of confidence region, for non-linear data. The main goal is to test the effectiveness of these methodologies using simulated data. The future goal is to use the proposed methodologies to study the fMRI data and get new insights.

Proposed Methodologies

We propose two methodologies-

1) The first approach is to use the so-called log-quantile density (Petersen & Mueller, 2016) to transform the non-linear data into linear data but apply recently developed tools in linear setting and then apply the inverse transformation.
2) The second approach is to use a geometric map, called log-transformation, to transform the non-linear data, then proceed as in the first approach.

Preliminary Results

In this project, we studied how to construct a confidence region for the "mean" of non-linear data. The implementation of methodologies were divided into three stages: i) map the non-linear data into linear data, ii) construct a confidence region of the transformed data in the linear setting, and iii) use an inverse map to go back to the non-linear setting. In this project, we considered simulated data consisting of probability density functions supported on a compact interval. We successfully implemented stages (i) and (ii) for the first approach where we used the so-called log-quantile density map for the stage (i). The implementation of (iii) and the second approach is

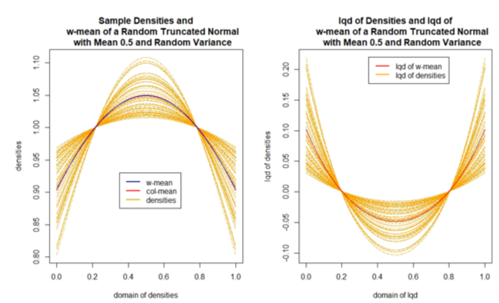

FIGURE 1. The figure on the left consists of sample densities drawn from $TN(0.5, \sigma^2)$ truncated on $[0,1]$ with $\log(\sigma) \in \mathcal{U}[-0.5.0.5]$. The figure on the right consists of the sample paths of lqd transformation of the truncated normal densities.

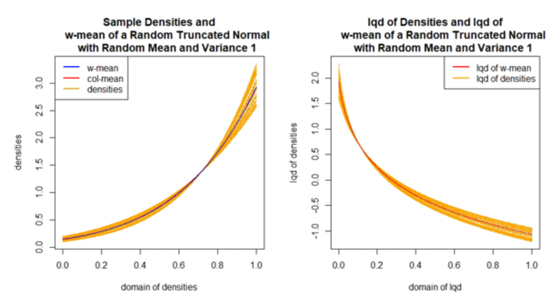

FIGURE 2. The figure on the left consists of sample densities drawn from $TN(\mu, 1)$ truncated on $[0,1]$ with $\mu \sim \mathcal{U}[3,4]$. The figure on the right consists of sample paths of lqd transformation of the truncated normal densities.

We compared the coverage probability; they are closer to 1-α which suggests that stage (i) and stage (ii) give satisfactory results for the numerical experiment. These preliminary results signal an optimism for the first approach.

Dissemination and Future Plan

I have written an abstract to present the result at Joint Mathematics Meeting 2024 in San Francisco. Two students worked with me on this project. Student researchers learned R programming and conducted numerical experiments using R. In the future, we plan to examine the efficacy of the second approach proposed in the project through simulation studies.

Selected References

Hani, H. S., Ata, E., Abbas, B.-F. (2019). Identification of the early stage of Alzheimer's disease using structural MRI and resting-state fMRI. Frontiers in Neurology, 10.

Petersen, A., and Mueller, H.-G. (2016). Functional data analysis for density functions by transformation to Hilbert space. Annals of Statistics, 44, 183-218.