Kinematics – Particle Linear Motion – Position and Velocity

Reminders: Velocity is the rate of change of _____ with respect to _____. A particle's speed is the _____ of that particle's velocity.

Notation: For particles moving along an axis.

- ✓ O = origin, s = position along axis, t = _____
- ✓ For a trip or time interval, Δs = particle displacement.
- ✓ If motion is horizontal (or predominantly so), "x" may be used for position.
- ✓ If motion is vertical, "y" may be used for position.

1. In each case, i. complete the chart ii. draw an axis, indicate the origin and - for each designated time - show the particle at its associated position. ii. Graph position vs. time and velocity vs. time; "stack" the graphs iii. Find the total displacement

a) Particle velocity is a constant -5 feet/sec

t (secs)	0	1	2	2.4	4.8	6
x (ft)	24					

b) Particle velocity is a constant + 4 meters/sec for $0 \le t \le 3$ seconds, a constant – 2.5 meter/sec for $3 < t \le 9$ seconds.

t (secs)	0	1	3	4	7	9
x (m)		0				

c) Particle moves downward. Make up the positive direction. Make y = 0 ground level. Particle moves with terminal velocity -64 feet/sec.

t (secs)	0	1	2.5	4	5
y (ft)	320				

2. Reminder: For an interval, **average velocity** = v_{av} = displacement over time interval = $\Delta s/\Delta t$). For each successive time interval, find the average velocity. Example:

t (secs)	0	1	2	3	5
x (ft)	0	2	8	18	50

- ✓ For t = 0 to 1 s, v_{av} = +2 ft/sec
- ✓ For t = 1 to 2 s, v_{av} = +6 ft/sec
- ✓ For t = 2 to 3 s, v_{av} = +10 ft/sec
- ✓ For t = 3 to 5 s, v_{av} = +16 ft/sec

a)

t (secs)	0	1	2	3	4
y (ft)	90	110	90	50	0

b)

t (secs)	0	0.8	0.81	0.82	1
s (m)	-10	0	1	1	-6

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3. (Reminder: Since instantaneous velocity is the rate of change of position, v(t) is equal to (circle one)

○ the derivative ○ the integral

of the position function s(t).

4. In each case i. determine the particle initial and final positions and at least 2 intermediate positions ii. draw an axis, indicate the origin and show the particle at its initial and final positions at least 2 intermediate positions iii. find the velocity function v(t) iv. at each drawn position, indicate the particle's velocity (magnitude and direction) iv. Graph position vs. time and velocity vs. time; "stack" the graphs.

a) $x(t) = -4 + 20*t - 2*t^2$ meters for $0 \le t \le 5$ seconds

b) $y(t) = 192 + 64*t - 16*t^2$ feet for $0 \le t \le 6$ seconds

5. Basic differentiation and chain rule practice. In each case, give the derivative of the given function. Don't forget the chain rule! *Example:* $x(t) = u^4$. *Answer:* $x'(t) = v(t) = 4*u^3 *du/dt$.

a) $x(t) = (3t)^3$

b) $y(t) = 4 \sin t + 8$

c) $x(t) = 3 \cos(\pi*t)$

d) $x(t) = 16 \sin(u) + 5$

e) $x(t) = 5*e^t$

f) $x(t) = 20 - 20*e^{-2t}$

g) $y(t) = \ln t$

h) $s(t) = 5 *\ln(t^2)$

6. In each case i. determine the particle initial position and at least 3 additional positions ii. draw an axis, indicate the origin and show the particle at its initial position and several other positions iii. find the velocity function v(t) iv. at each drawn position, indicate the particle's velocity (magnitude and direction) v. Graph position vs. time and velocity vs. time; "stack" the graphs.

a) $x(t) = 220 - 220*e^{-0.2t}$ feet, $t \ge 0$ seconds. *(Scenario: Vehicle cruising to a stop due to laminar air resistance)*

b) $y(t) = -128 + 64*t + 128* e^{-0.5t}$ feet, $t \ge 0$ seconds. Make down the positive direction. (Scenario: Free-fall with laminar air resistance)

7. (Reminder) Since instantaneous velocity is the rate of change of position, s(t) is equal to (circle one)

○ the derivative ○ the integral

of the velocity function v(t).

8. In each case, write the position function.

a) $v(t) = 2t + 4$ meters/sec; at time $t = 0$, the particle is at $x = -5$ meters.

b) $v(t) = 5 \cos(2.5\, t)$ feet /second; at time $t = 0$ seconds, the particle is at $y = 10$ feet.