## Rotational Kinematics

For a rotating object, instead of thinking about s(object position), we start with
$\checkmark \boldsymbol{\theta}$ (angular position)
$\checkmark \boldsymbol{\theta}_{\mathrm{o}}$ (angular position at the start of a time period) and
$\checkmark \Delta \boldsymbol{\theta}$ (angular displacement or rotation for a time period).
$\checkmark$ The text uses $\mathbf{N}$ for angular displacement or rotation expressed in revolutions.


1. Convert the following rotations to radians and revolutions. Recall that $2 \pi$ rads $=360^{\circ}$.
a. 540 degrees clockwise
b. 90 degrees CCW
c. 572.95 degrees CW

For rotation rate or angular velocity, use the symbol $\boldsymbol{\omega}$. For units, we usually use radians/sec and rpm.
2. a. Convert 6.28 radians/sec to rpm
b. Convert 3000 rpm to radians/sec

Since $\boldsymbol{\omega}$ is a rate, it can be defined as a derivative; $\boldsymbol{\omega}=\mathbf{d} \boldsymbol{\theta} / \mathbf{d t}=\dot{\boldsymbol{\theta}}$
3. A wheel's rotation is given by $\theta(t)=32 t-2 t^{2}$ radians for $t \geq 0$.
a. Find an expression $\omega(\mathbf{t})$ for the wheel's rotation rate.
b. Find the rotation rate at $t=2$ and $t=6$ seconds.
c. Recall that, by the right-hand rule, CCW is considered the positive direction. Find the magnitude and direction of the rotation at $t=10$ seconds.
d. At what time(s) does the wheel change rotational direction?
4. At time $t=0$, a wheel's motor is shut off and the rotation is given by $\omega(t)=63 e^{-0.2 t} \mathbf{r a d i a n s} / \mathbf{s e c}$.
a. Complete the following chart

| $\boldsymbol{t}$ (secs) | 0 | 2 | 4 | 8 | 20 |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\omega$ (rpm) |  |  |  |  |  |

b. Use integration to find the wheel's rotation $\Delta \boldsymbol{\theta}$ in radians and $\mathbf{N}$ in revolutions after i. 2 seconds ii. 4 seconds
c. (Bonus). Find the total number of revolutions by the wheel from the instant the motor is shut off until is stops rotating.

Angular acceleration is the rate that the angular velocity is changing. Use the symbol $\boldsymbol{\alpha}$. For units, we usually use radians/sec change in rotation per second or rad $/ \mathrm{sec}^{2}$.
5. In each case determine average $\boldsymbol{\alpha}$ in rad/ $\mathrm{sec}^{2}$. You can compute using $\Delta \omega / \mathrm{t}$.
a. A shaft's rotation rate increases from $2 \mathrm{rad} / \mathrm{sec}$ to 628 radians $/ \mathrm{sec}$ in 4.5 seconds.
b. A shaft's rotation rate decreases from 3000 to 60 rpm in 10 seconds.

