Math 12 – Extra Problems

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1. Write $\dfrac{7000000 \cdot (200)^3}{42000}$ in scientific notation.

2. Simplify by writing with positive exponents $\left(\dfrac{2b^{-2}a^2}{b^3a^4}\right)^{-2}$ assuming no variables are zero.

3. Solve for b
$$\frac{3(b+2a)}{6} = b - 7$$

4. Runner runs a total of 126 miles in 6 days. Their training regimen requires that each day they run twice as far as the day before. How far did they run on each day?

5. An artist has a painting that is 3 ft by 2 ft. They want to make a frame that has uniform width. The artist wants the framed painting to have a perimeter of 13 feet, how wide should the frame be.

6. Consider the points $(3, 4)$ and $(5, 7)$.
   (a) Find the equation of the line that passes through both points.
   (b) Find a perpendicular line which passes through the midpoint of $(3, 4)$ and $(5, 7)$.
   (c) Find a a line parallel to the line in part (a) which passes through the point $(3, 7/2)$

7. Let $f(x) = 3x - 2$ and $g(x) = 2x^2 - 1$.
   (a) Find $g(1) + f(-2)$
   (b) Find $(f(-1))^2 \cdot 3 - 2 \cdot g(-1)$
   (c) If $f(x) = 9$, what is the value of x?

8. Graph the line $\dfrac{4x - 2y}{3} = 5$, identify the slope and the y-intercept.

9. If possible, solve the systems
   (a) $\begin{cases} 2x + 3y = 1 \\ x + 3y = 2 \end{cases}$
   (b) $\begin{cases} \frac{2}{3}x + \frac{1}{2}y = \frac{1}{2} \\ 2x - y = 1 \end{cases}$
   (c) $\begin{cases} 4x + y = 6 \\ \frac{2}{3}x + \frac{1}{6}y = 1 \end{cases}$
   (d) $\begin{cases} x - y = 6 \\ -2x + 2y = 1 \end{cases}$

10. Solve for x. Give your answer as a graph and in interval notation
    (a) $3x + 2 \geq -\left(2x - \frac{1}{2}\right)$
    (b) $\frac{x}{5} + 1 \geq 4$ or $\frac{1}{7}(x + 1) - 1 < 3$

11. Solve the absolute value inequalities, express your answer in interval notation and on the number line
    (a) $\left|\dfrac{2}{3}x - \dfrac{1}{2}\right| - 3 > 1$
    (b) $\dfrac{1}{2}\left|\dfrac{1}{3} - x\right| - 2 \leq 7$

12. Graph the solution set of the inequalities $\begin{cases} y > 2x - 1 \\ y \leq 1 - \frac{1}{2}x \end{cases}$

13. Write each number in scientific notation and simplify without a calculator

    (a) $2,150,000,000$

    (b) $(0.0000035)^2$

    (c) $(2,000,000,000,000)^5$

    (d) $\dfrac{300 \cdot 2800}{70}$

    (e) $\dfrac{(100)^{10} \cdot 4800}{80}$

    (f) $\dfrac{0.0000002 \cdot (42,000,000,000)^2}{2100}$

14. Graph the following lines

    (a) $2x - y = 1$

    (b) $y = 4x + 2$

    (c) $x - 2 = 0$

    (d) $y = 7$

15. This table gives the cost of producing an item,

Items manufactured	1	2	3	4	5
Cost in Dollars	427	428.75	430.50	431.75	433.50

    When graphed, does this data form a line or not?

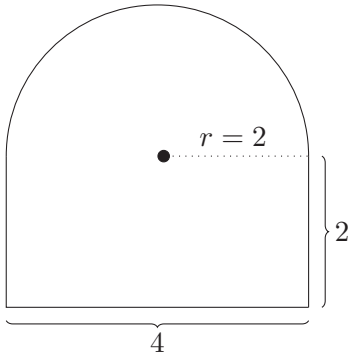
16. A student's homework points in a class are scaled to 200 points of their total grade. If the student has 71 out of 101 points possible before scaling, how many points will they have after scaling?

17. There are three laps in a race. A runner completes their first lap in 1 minute and 56 seconds, their second lap in 1 minute and 35 seconds, and their final lap in 1 minute and 50 seconds. What was the average of the three laps?

18. A student has an average of 33 points over three quizzes, what is the sum of their three quiz scores?

19. A student has an average of 81% for their three midterm exams. On the first exam they received 91% and on the third they received a 76%. Unfortunately the student cannot locate their second exam, what score did they get?

20. Find the area and perimeter of the following figure:



21. A student received an 86 points on their second midterm exam in a class. This was a 15% increase from their first exam. How many points did the student get on their first exam?

22. For each function decide if y is a function of x

    (a) $2x + y = 7$ (b) $x^2 + y^2 = 1$ (c) $y = 2x + 3y$

23. A student takes three midterm exams and a final. Their overall exam percentage is the average of the percents on all four exams. They score a 59% on the first exam an 89% on the second and a 79% on the third. What average do they need on the final to score above an 80%?

24. Graph the following functions to find their intersection points.

    (a) $f(x) = |x|$ and $g(x) = 2x + 3$
    (b) $f(x) = |x|$ and $g(x) = \frac{1}{2}x + 3$
    (c) $f(x) = |x|$ and $g(x) = \frac{1}{2}x - 2$

25. Usain Bolts world record run of the 200 meter dash in 2009 was 19.19 seconds.

    (a) What was his average speed on his record breaking run?
    (b) Roughly how many meters does Usain run in 1 second? 2 seconds? 3 seconds? x seconds?
    (c) Suppose your PAL facilitator runs the 200 meter dash at an average speed of 5.5 meters per second. If they are given a 15 second head start after how many seconds would Usain catch up with them?
    (d) Who would win the race?

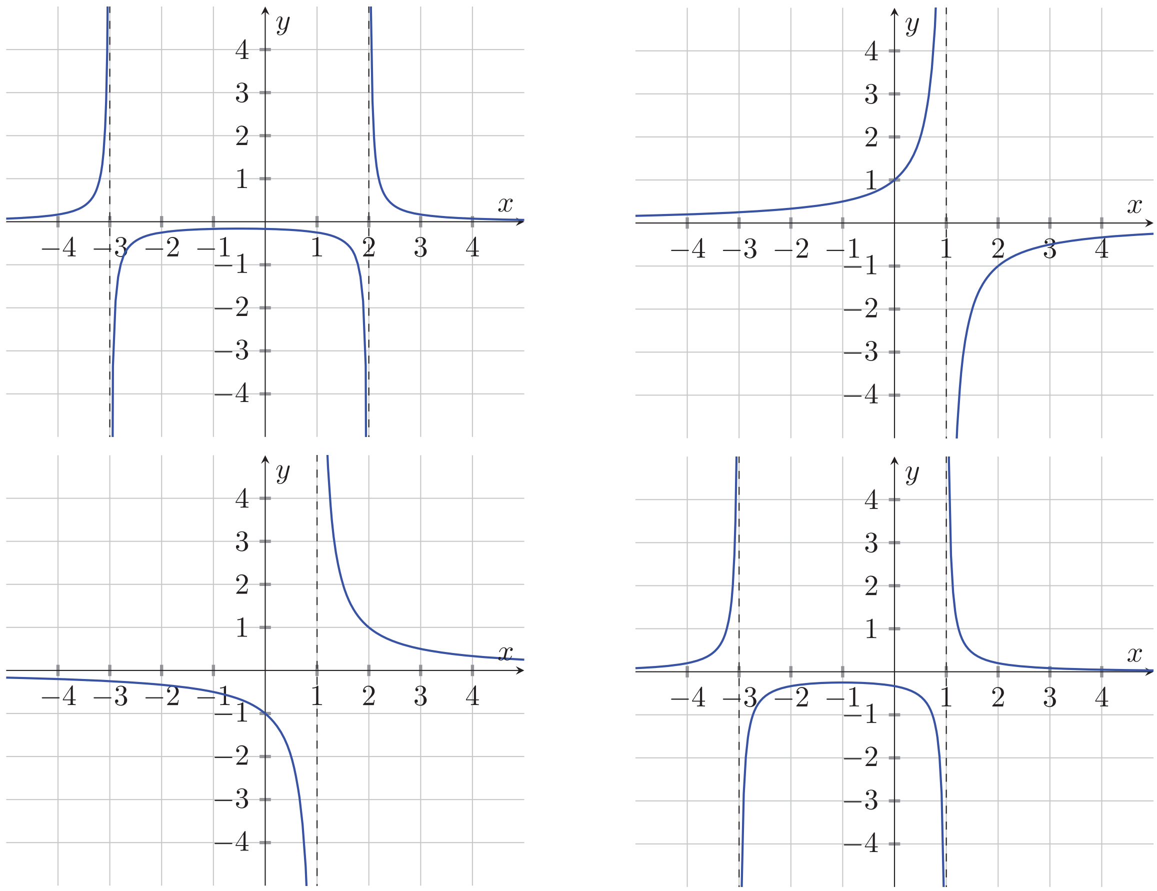
26. Suppose a raid boss in a video game has 2.1 million health. Your raid group does an average of 300 thousand damage per minute.

    (a) Create a linear function which describes the bosses health after x minutes.
    (b) How long does it take your raid group to defeat the boss?
    (c) Suppose that 4 minutes into the fight your two best players disconnect, reducing your average damage to 190 thousand damage per minute. Can you still win the fight before the 10 minute enrage timer?

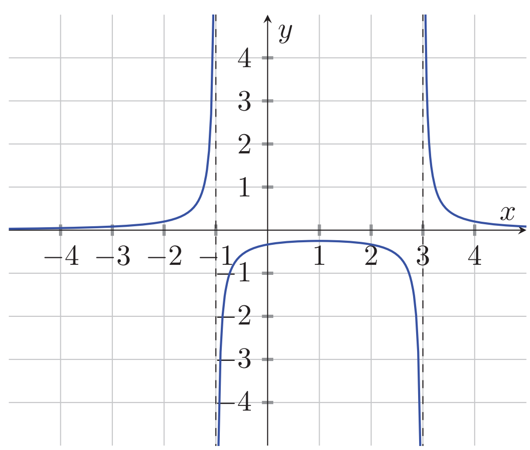
27. A rectangle has width 3 inches longer than twice it's length. Ignoring units, the area of the rectangle is 3 less than it's perimeter. What are the rectangles dimensions?

28. Find the domains of the rational functions, then match the functions with their graphs.

- $f(x) = \dfrac{1}{x-1}$
- $g(x) = \dfrac{1}{1-x}$
- $h(x) = \dfrac{1}{(x-2)(x+3)}$
- $j(x) = \dfrac{1}{(x^2+2x-3)}$



29. Use the information in problem (2) to create a function which *could* have the following graph:



30. Mutiply out the following and simplify the exponents

(a) $(x^{-1} + y^2)(y^{-2} - x)$

(b) $\left(3x^2 + 2x - \dfrac{1}{x^3}\right) x^{-1}$

31. Simplify simplify and write with positive exponents

(a) $\left(\left(\dfrac{2 \cdot 81 \cdot x^{-3} z^4 y^{-1}}{x^{-2} z^3 \cdot 3^4 \cdot y^{-2}}\right)^2\right)^{-3}$

32. A triangle is right if its sides satisfy the Pythagorean Theorem. Is the triangle formed by connecting the points $(0,1)$, $(3,7)$ and $(4,-1)$ a right triangle?

33. Give a point with y coordinate 12 which is 13 units away from $(0,7)$

34. Give an x value for which $g(x) = \dfrac{(x+2)(x-1)}{(x+2)}$ and $h(x) = x - 1$ are differet.

35. Simplify

$$\frac{x + 1 - \frac{6}{x}}{\frac{1}{x}}.$$

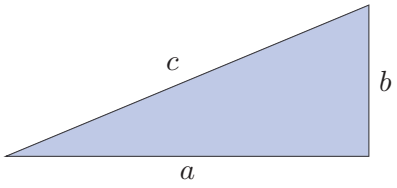
For which x is the above expression zero?

36. Simplify the following

(a) $\sqrt[3]{\dfrac{8y^{-9}}{z^{15}x^{-3}}}$

(b) $\sqrt{x^2 - 12x + 36}$

37. Consider the right triangle with side lengths a, b and c as shown



The below squares were both constructed with four copies of the above right triangle

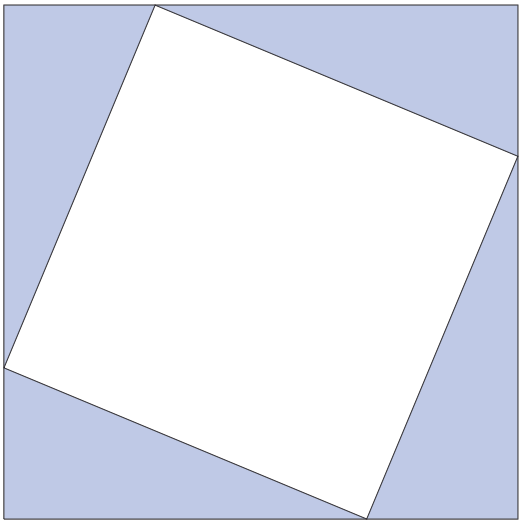


figure 1

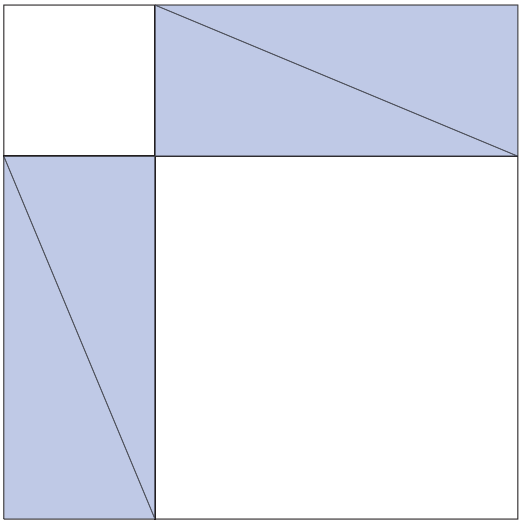


figure 2

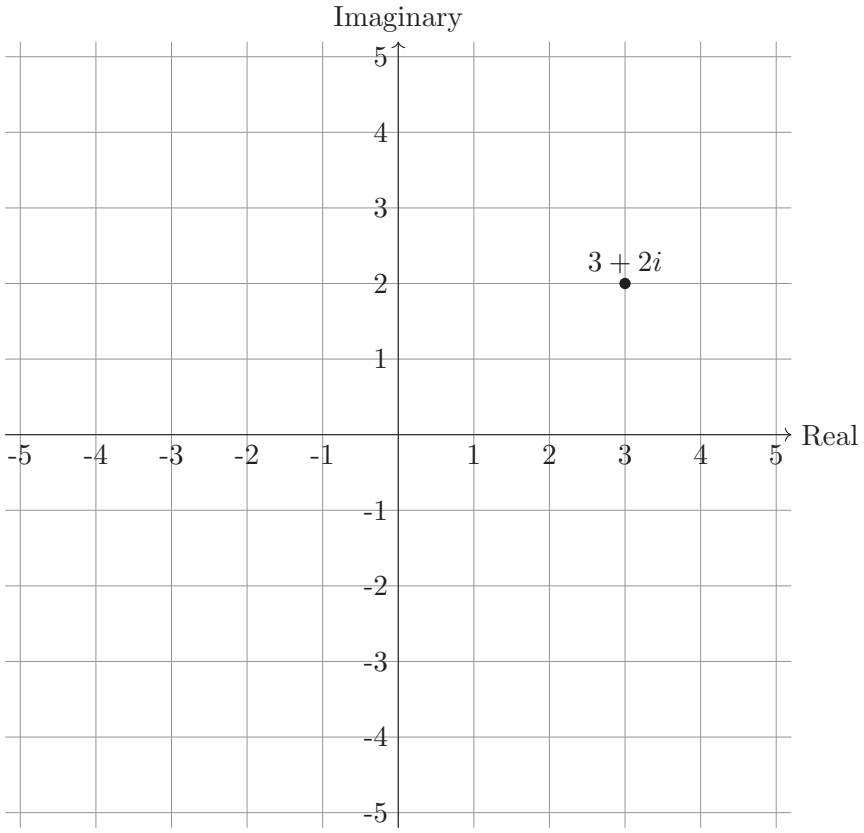
(a) What is the area of figure 1 and figure 2 in terms of a, b and/or c?

(b) Without any calculation, how do the shaded areas of figure 1 and 2 compare? What does this tell us about the area of the unshaded portions of the figures?

(c) Compute the area of the unshaded regions of figure 1 and 2 in terms of a, b and c, and make a conclusion using part b.

The following was a worksheet over the geometry of complex numbers:

38. We can think of a complex number as a point on a plane, for instance we can think of the complex number $3+2i$ as a the point $(3,2)$. To plot this point move three units to the right on the x axis which we will call the real axis, and 2 units up on the y axis which we will call the imaginary axis. Below we see $3+2i$ plotted



39. Plot the given complex numbers

(a) $2+i$

(b) $3i$

(c) 3

(d) $-3+3i$

(e) $1-i$

40. Plot each complex number and find its distance from the point $(0,0)$

(a) $3+4i$

(b) $24-7i$

(c) $1+\sqrt{3}i$

41. We can define the absolute value of a complex number as its distance from $(0,0)$ when plotted. Find the following

    (a) $|3i|$
    (b) $|3-6i|$
    (c) $|5+7i|$
    (d) $\left|\frac{1}{2}-\frac{\sqrt{3}}{2}i\right|$
    (e) $|a+i|$
    (f) $|2+bi|$
    (g) $|a+bi|$

42. Draw a graph of all complex numbers which have absolute value 1.

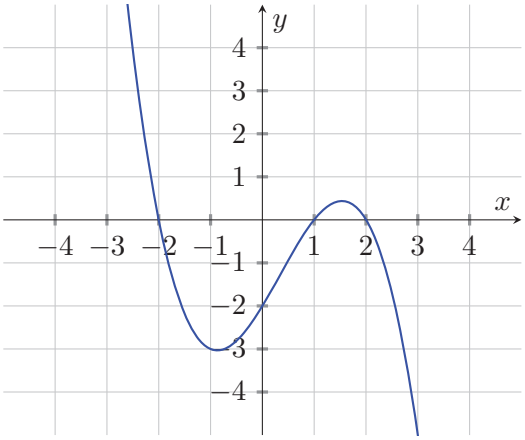
43. Plot the points

    - $4+i$
    - $i\cdot(4+i)$
    - $i^3\cdot(4+i)$
    - $i^4\cdot(4+i)$

    and make a conjecture about what happens graphically to a complex number when you multiply it by i.

    END of geometry of complex numbers workshop

44. Consider the graph of $f(x)$



    (a) What are the y intercepts of $f(x)$?
    (b) What are the x intercepts of $f(x)$?
    (c) On which intervals is $f(x)>0$?
    (d) On which intervals is $f(x)\leq 0$?

45. Let $g(x)=x^2-3x+2$

    (a) What are the zeros of $g(x)$? What is the y intercept?
    (b) On which intervals is $g(x)\geq 0$
    (c) On which intervals is $g(x)<0$

46. Find the solution set of each of the following. Write your answer with inequalities, on the number line and in interval notation.

    (a) $x^2-7x+12\geq 0$
    (b) $4x^2-4x+1<0$
    (c) $\dfrac{1}{x}-3<4$

(d) $(x-1)(x-2)(x-3) < 0$

(e) $(x-1)(x-2)(x-3)(x-4)^2(x-5) > 0$

47. On which intervals is
$$\frac{(x+1)^2 - 2x(x+1)}{(x+1)^4}$$
negative? On which intervals is it greater than or equal to zero?

48. Suppose you have an investment of 10,000 dollars invested at a rate of 4.5 % compounded twice a year. How much money will it be worth in 20 years? How much would your investment be worth after 20 years if interest was compounded continuously?

49. Factor Completely

(a) $256x^8 - 1$

(b) $a^2b + a - 2 - 2ab$

50. Solve

(a) $2y^2 + 4y - 47 = 1$

(b) $16x^3 - 50x^2 = -36x$

51. A grasshopper jumps straight up with an initial velocity of 16 feet per second from a tree 32 ft above the ground. The height of the grasshopper at time t is given by $h(t) = -16t^2 + 16t + 42$. How long before the grasshopper is 10 feet off the ground?

52. What values of x make the following rational function undefined?
$$f(x) = \frac{3x+1}{2x^2 + 11x + 12}$$

53. Preform the operation and then simplify:

(a) $\dfrac{x+1}{x-2} - \dfrac{x-1}{x+2}$

(b) $\left(\dfrac{x^2-9}{x^2+3x+yx+3y}\right) \div \left(\dfrac{x-3}{x+3}\right)$

54. Simplify
$$\frac{\frac{1}{x^2} - \frac{1}{x}}{\frac{1}{x} - \frac{1}{x+1}}$$

55. Solve
$$\frac{3}{x-2} - \frac{1}{x-1} = \frac{7}{x^2 - 3x + 2}$$

56. Divide $6x^3 + 3x^2 - 6x + 5$ by $2x - 3$ using polynomial long division.

57. Use the fact that $\sqrt{y^2} = |y|$ to solve the following
$$\sqrt{4x^2 - 12x + 9} \leq 4$$

58. Simplify and write with positive exponents
$$\left(\frac{2^4 x^{-1} y^{\frac{2}{3}}}{x^{-2} y^3}\right)^{-2}$$

59. Suppose that a rectangle has a longer side that is twice the length of the shorter side. Suppose both side lengths were increased by 2 units which makes the area 3 times larger. What was the area of the square before the sides were lengthened. Express your answer as a complete sentence.

60. (a) Find the distance between the two points $(1, 2)$ and $(5, 7)$
    (b) Find three other points which are the same distance from $(1, 2)$ as $(5, 7)$.

61. Determine if the following statements are always true. If yes, clearly explain why. If no, pick numbers for x and y to provide a counter example.
    (a) $\sqrt{x^2 + y^2} = x + y$
    (b) $(\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$

62. Simplify
$$\sqrt{81x^3y^2} + 2\sqrt{24xy^3} + y\sqrt{3x^5y}$$

63. Rationalize the denominator and simplify
$$\frac{\sqrt{10x^2y}}{\sqrt{2x^3y}}$$

64. Rationalize the denominator and simplify
$$\frac{x - 22}{\sqrt{\frac{1}{2}x - 2} - 3}$$

65. Solve
$$\sqrt{x} = \frac{4}{\sqrt{x + 6}}$$

66. Solve for x using any method
$$\frac{2}{x + 3} + \frac{12}{x + 2} = 11$$

67. Solve $x^{-2/3} - 2x^{-1/3} - 3 = 0$. Hint $x^{-2/3} = (x^{-1/3})^2$