Math 31 – Workshop #28

1. Consider the function $f(x) = \sin x$.
   (a) Find the first four non-zero terms of the Taylor series for $f(x)$ centered at $a = \frac{\pi}{2}$.
   (b) Write the Taylor series for $f(x)$ centered at $a = \frac{\pi}{2}$ in summation notation.
   (c) Let $T_4(x)$ be the polynomial we get if we remove all terms with degree higher than 4 in the Taylor series. Graph $T_4(x)$ and $f(x)$ on your graphing calculator. For what values of x would you say $T_4(x)$ is a good estimate of $f(x)$?
   (d) Now graph $T_6(x)$ on the same set of axes as $f(x)$. For what values of x would you say $T_6(x)$ is a good estimate of $f(x)$?

2. Consider the function $f(x) = \ln x$.
   (a) Find the first five non-zero terms of the Taylor series for $f(x)$ centered at $a = 1$.
   (b) Write the Taylor series for $f(x)$ centered at $a = 1$ in summation notation.
   (c) Use $T_2(x)$ to estimate $\ln 1.1$.
   (d) Plug $\ln(1.1)$ into your calculator. To how many decimal places was your estimate correct?
   (e) If you want your estimate of $\ln(1.1)$ to be correct up to 6 decimal places, how many terms of the Taylor series would you need?

3. (a) Find the degree three Taylor polynomial for $f(x) = \sqrt[3]{x}$, centered at $a = 1$.
   (b) Use this to estimate $\sqrt[3]{0.5}$.

4. Find the area of the region bounded by $y = \sqrt{x-1}$ and $y = x - 1$.

5. Determine whether the following integrals converge or diverge. If they converge, compute them.
   (a) $\displaystyle\int_1^{\infty} \frac{1}{(2x+1)^3}\,dx$
   (b) $\displaystyle\int_{-\frac{1}{2}}^{1} \frac{1}{(2x+1)^3}\,dx$
   (c) $\displaystyle\int_{-1}^{-\frac{1}{2}} \frac{1}{(2x+1)^3}\,dx$
   (d) $\displaystyle\int_{-\infty}^{-1} \frac{1}{(2x+1)^3}\,dx$