# Math 32 – Workshop #15

1. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

   (a) $x^4 + 8y^3 - z^2 = 4$

   (b) $8y^3x^4 - e^z = 4$

2. A string is stretched along the $x$-axis, fixed at each end, and then set into vibration. It is shown in physics that the displacement $y = y(x,t)$ of the point of the string at location $x$ at time $t$ satisfies the one-dimensional wave equation
$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2},$$
where the constant $a$ depends on the density and tension of the string. Show that $y = \sin(kx)\cos(kat)$, where $k$ is a constant, satisfies this wave equation.

3. Find the equation of the tangent plane to the surface $f(x,y) = \frac{x}{x+y}$ at $(2,1)$. From this, find the linearization $L(x,y)$ of $f(x,y)$ at $(2,1)$.

4. Verify the linear approximation $\frac{2x+3}{4y+1} \approx 3 + 2x - 12y$ at $(0,0)$.

5. Use differentials to estimate the amount of metal in a closed cylindrical can that is 8 cm high and 4 cm in diameter if the metal in the top and bottom is 0.2 cm thick and the metal in the sides is 0.05 cm thick.

6. The length and width of a rectangle are measured as 24 cm and 16 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.