Math 32 – Workshop #4

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1. Which of the following expression are meaningless (i.e. not defined)? For those that are meaningful (i.e. defined), state whether the expression is a scalar or a vector.

   (a) $|\vec{\mathbf{v}}| \times \vec{\mathbf{w}}$

   (b) $(|\vec{\mathbf{v}}|\vec{\mathbf{v}}) \times \vec{\mathbf{v}}$

   (c) $(\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}) \times \vec{\mathbf{w}}$

   (d) $(\vec{\mathbf{v}} \times \vec{\mathbf{v}}) \times \vec{\mathbf{w}}$

2. We have two vectors in $\mathbb{R}^3$, $\vec{\mathbf{v}} = \langle 1, 3, 2\rangle$ and $\vec{\mathbf{w}} = \langle -1, -1, 4\rangle$. Find three vectors that are perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$. How many unit vectors are there that are perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$? Find all such unit vectors.

3. Set $\vec{\mathbf{v}} = \langle v_1, v_2, v_3\rangle$ and $\vec{\mathbf{w}} = \langle w_1, w_2, w_3\rangle$. Verify the property $(2\vec{\mathbf{v}}) \times \vec{\mathbf{w}} = 2(\vec{\mathbf{v}} \times \vec{\mathbf{w}})$. Also verify the property $(2\vec{\mathbf{v}}) \cdot \vec{\mathbf{w}} = 2(\vec{\mathbf{v}} \cdot \vec{\mathbf{w}})$. Work from left to right for verifications. *(These are examples of the general properties $(c\vec{\boldsymbol{v}}) \times \vec{\boldsymbol{w}} = c(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}})$ and $(c\vec{\boldsymbol{v}}) \cdot \vec{\boldsymbol{w}} = c(\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{w}})$, where c is a scalar. These properties allow us to simplify computations.)*

4. Set $\vec{\mathbf{v}} = \langle 1, 3, 2\rangle$ and $\vec{\mathbf{w}} = \langle -1, -1, 4\rangle$, and let θ be the angle between these two vectors.

   (a) Find an expression for $\cos\theta$ and also for $\sin\theta$.

   (b) What is the quickest way to determine if these two vectors are perpendicular?

   (c) What is the quickest way to determine if these two vectors are parallel?