**Stat 50 – Worksheet #10: The Binomial Distribution**

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1. In a very large shipment of packaged fresh spinach, 10% of the packages are contaminated with the bacteria *E. Coli*. Suppose a Food and Drug Administration inspector randomly chooses 3 packages of spinach to test for *E. Coli*.

   (a) We can represent an outcome of this experiment as a sequence of 3 S's and F's where S means *E. Coli* was found in a package of spinach and F means no *E. Coli* was found in the package. For example, the outcome SFS means the first and third packages of spinach had *E. Coli* while the second did not have *E. Coli*. Also, SSS means all 3 packages of spinach had *E. Coli*. Use a tree diagram or other method to list the 8 possible outcomes of this experiment.

   (b) Let the random variable $X$ = the number of packages out of the 3 inspected that had *E. Coli*. Complete the blank cells in the table below with the outcomes in $\mathcal{S}$ corresponding to each value of $X$.

| X | outcomes in $\mathcal{S}$ |
|---|---|
| 0 | |
| 1 | SFF, FSF, FFS |
| 2 | |
| 3 | SSS |

   (c) What is $P(SFF)$? Use the fact that $P(X = 1) = P(SFF) + P(FSF) + P(FFS)$ to compute $P(X = 1)$.

   (d) Use similar reasoning from the last part to complete the probability distribution of $X$ below:

| X | outcomes in $\mathcal{S}$ | $P(X = x)$ |
|---|---|---|
| 0 | | |
| 1 | SFF, FSF, FFS | answer to 1(c)____ |
| 2 | | |
| 3 | SSS | |

   (e) What is the probability *E. Coli* will be detected in at most two of the sampled packages?

   (f) Calculate $\mu_X$ two ways: 1) using $\mu_X = \sum_x x \cdot p(x)$ and 2) using the special formula for the mean of a binomial random variable, $\mu_X = np$. Compare your results.

2. A coin is biased so that the probability of heads is 0.7. Let $X$ = the number of heads in 5 tosses of this coin. A student calculates $P(X = 1)$ as follows:

$$P(X = 1) = P(SFFFF) = 0.7(0.3)^4$$

   (a) Explain the flaw in the students calculation and show the correct calculation of $P(X = 1)$.

   (b) What is the minimum number of coin flips needed to have at least 0.999 probability of getting at least one heads?

(c) If the coin is tossed 10 times, how many outcomes with exactly 4 heads exist? Write out 3 outcomes with exactly 4 heads in 10 tosses.

3. A fair six-sided die is tossed 600 times. Let $X$ = the number of tosses on which 2 appears face up.

   (a) Calculate $\mu_x$ and $\sigma_x$

   (b) Calculate the interval $\mu_x \pm 2\sigma_x$.

   (c) According to Chebychev's Rule what is the minimum probability that $X$ will fall in the interval from part (b)?

   (d) Would it be unusual to observe the value 2 face up 150 times out of the 600 tosses? (Hint: The last part will help.) Justify your answer.