Stat 50 – Worksheet #11: The Poisson Distribution

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1. Find the value of the following sums:

   (a) $\displaystyle\sum_{x=0}^{\infty} \frac{e^{-2}2^x}{x!}$

   (b) $\displaystyle\sum_{x=2}^{\infty} \frac{e^{-2}2^x}{x!}$

2. The number of cactus plants in a square yard of land in a certain desert has a Poisson Distribution with a mean of 0.2 cactus per square yard.

   (a) What is the probability there are between 1 and 3 (inclusive) cactus plants in a square yard?

   (b) What is the probability there is at most one cactus plant on a randomly selected square yard?

   (c) Give the probability mass function for the number of cactus plants in 10 square yards of land in this desert.

   (d) What are the mean and standard deviation of the number of cactus plants in a 10 square yard plot of land in this desert.

   (e) What is the probability of at least 2 cactus plants in a random 10 square yard plot in this desert?

3. Suppose the random variable X has a Poisson distribution with parameter λ, where the value of λ is unknown. However, it is known that $P(X = 0) = 2P(X = 1)$. Find λ. (Hint: Write out $P(X = 0)$ and $P(X = 1)$ using the formula for Poisson probabilities even though λ is unknown.)

4. For a binomial random variable, if n is large and p is very small, then the Poisson distribution with $\lambda = np$ will provide a very good approximation to the binomial distribution. In this exercise, we will compare a Poisson approximation to a binomial probability to the exact binomial probability. Suppose X has a binomial distribution with $n = 1000$ and $p = 0.002$

   (a) Use the binomial probability formula to calculate $P(X = 1)$ exactly.

   (b) Use the Poisson distribution with $\lambda = np$ to approximate the probability in part (a).

   (c) Compare your answers in parts (a) and (b). Is the Poisson a good approximation to the binomial in this case?

   (d) Now suppose, $X \sim Bin(n = 3, p = 0.5)$. Calculate the exact binomial probability that $X = 1$ as well as the Poisson approximation to this binomial probability (note that $\lambda = np = 3(0.5)$) and compare your answers. (As a rule fo thumb, the Poisson approximation will be very good if $n \geq 100$ and $np \leq 10$. So the approximation is not quite so accurate in this part.)