Stat 50 – Worksheet #19 and #20: Review of Discrete and Continuous Probability Distributions and Central Limit Theorem

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1. Suppose the duration of a country music song is normally distributed with mean 3.5 minutes and standard deviation 0.2 minutes.

   (a) What percent of country music songs are over 3.9 minutes long?

   (b) What percent are between 3.4 and 3.9 minutes long?

   (c) Find the 30th percentile of lengths of country music songs.

   (d) What would the standard deviation need to be so that the probability a country music song exceeds 3.7 minutes is 0.40?

   (e) If 80 country music songs are randomly selected, what is the probability the **sample mean duration** is under 3.55 minutes?

   (f) If 10 country music songs are randomly selected, what is the probability the total duration of the sampled songs is over 38 minutes long?

2. Suppose that 20% of Americans are runners.

   (a) If 5 Americans are randomly selected, what is the probability

      i. exactly three are runners

      ii. at least two are runners

   (b) If 500 Americans are randomly selected

      i. what are the mean and standard deviation of the number of runners among the 500 Americans selected

      ii. what is the probability at most 90 of the 500 are runners?

   (c) Letting S represent a runner and F a non-runner

      i. write out all possible sequences of S's and F's that would correspond to the event exactly two of five people are runners AND the last person is a runner.
      Hint: One such outcome is FSFFS, list all others.

      ii. What is the probability of the outcome FSFFS?

      iii. What is probability of obtaining your second runner on the fifth person sampled? (Hint: Use previous parts.)

3. Suppose X, the number of knots in yard of rope, has a Poisson distribution.

   (a) If the probability of no knots in a yard is 0.25, find the mean of X.

   (b) Find $P(X \geq 2)$

   (c) Let the random variable $Y =$ the distance in yards between successive knots. What are the mean and standard deviation of the distance between knots?

   (d) What is the probability that the distance between knots is more than 2 yards?

   (e) Find the 60th percentile of distances between knots.

4. Suppose a fair six-sided die is tossed until the first 5 appears. What is the probability it takes

   (a) exactly 4 tosses?

   (b) at least 10 tosses?

   (c) What is the average number of tosses needed to obtain the first 5?

5. A fish tank at a pet store contains 15 goldfish and 7 koi. A customer is going to use a fishing net to randomly catch 4 fish from the tank to buy. Let X = the number of koi fish among the 4 the customer buys.

   (a) Find $P(X = 4)$

   (b) What is the probability the customer gets 4 fish of the same kind?

   (c) What is the formula for $P(X = x)$ for $x = 0, 1, 2, 3, 4$?

   (d) What is the average (or mean) number of koi that will be obtained by the customer?