Stat 50 – Worksheet #1: Review of Calculus

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1. Find the derivative of each function.
   (a) $f(x) = 2x^4 + \dfrac{1}{x^3} + x + 35$
   (b) $f(x) = 1 - e^{-5x} + \ln(x)$

2. Let g be the piecewise-defined function given below.

$$g(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \frac{1}{2}x & \text{if } 0 < x \leq 1 \\ \frac{3}{4} & \text{if } 1 < x \leq 2 \\ 0 & \text{if } x > 2 \end{cases}$$

   (a) Sketch a graph of g
   (b) Find $\int_0^2 g(x)dx$
   (c) Find $\int_{-\infty}^{\infty} g(x)dx$
   (d) Find $\int_{-\infty}^{1.5} g(x)dx$
   (e) Find $\int_{-\infty}^{-0.7} g(x)dx$ (Don't work too hard.)
   (f) Find $\int_{-\infty}^{k} g(x)dx$ where k is a real number with $k < 0$

3. Find the value of k that satisfies the following equation:

$$\int_0^1 kx^2(1-x)dx = 1$$

4. Find the following limits:
   (a) $\lim_{x\to\infty}(e^{-\frac{x^2}{2}})$
   (b) $\lim_{x\to-\infty}(e^{-x^2})$
   (c) $\lim_{x\to-\infty}(1 - e^{-2x})$
   (d) $\lim_{x\to-\infty} \dfrac{1}{\pi(1+x^2)}$

5. Recall that a geometric series of the form $\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \cdots$ converges to $\frac{a}{1-r}$ if $|r| < 1$. Determine if the given series converges. If the series converges, find its sum.
   (a) $\sum_{k=0}^{\infty}(\frac{\pi}{2})^k$
   (b) $\sum_{k=0}^{\infty}(\frac{1}{2})^k$
   (c) $\frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \cdots$ (Hint: What is a and what is r?)