Stat 50 – Worksheet #27: Mixed Practice Problems for Final Exam

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1. A new look-under-the-cap and win game is being promoted by Flobee Beverage Company. Flobee claims that 1 in 8 bottles of Thirstquencher Soda contain winning caps (you win a prize).

   (a) You and your friends drink 32 bottles of Thirstquencher Soda and no one wins a prize. What is the probability of this happening if the probability of winning really is 1/8?

   (b) What is the probability of winning 2 or more prizes from the 32 bottles?

   (c) What are the expected value and standard deviation of the number of winning caps in 32 randomly selected bottles?

   (d) If 1000 Thirstquenchers are bought, what is the probability the number of winning caps is less than 100?

2. Of 100 randomly selected people who were given Vaccine X, 88 develop immunity to the chicken pox.

   (a) Estimate the proportion of the population that will develop immunity when given the vaccine using a 95% confidence interval.

   (b) The company that sells the vaccine claims that 98% of people who get the vaccine will develop immunity to chicken pox. Does your confidence interval support or refute this claim? Give reasons for your answer.

   (c) Suppose an estimate of the proportion which will develop immunity with margin of error 0.03 is required. What is the minimum sample size needed,

      i. assuming you know that 88 out of 100 developed immunity?

      ii. Assuming no estimate of the proportion which develops immunity is available?

3. Ten years ago the silicon content of the water in a certain river was 5 mg/L. 85 water samples taken recently from the river have mean silicon content 5.6 mg/L and standard deviation 1.2 mg/L. Can you conclude that the silicon content of the water is greater today than it was 10 years ago?

   (a) State the null and alternative hypothesis.

   (b) Find the P-value.

   (c) At the 0.05 level of significance, would you reject the null hypothesis?

4. The weekly repair cost X (in hundreds of dollars) for a certain machine has a probability density function given by

$$f(x) = \begin{cases} 3(1-x)^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the mean and variance of X.

(b) Use Chebychev's Rule to find an interval where the weekly repair cost will fall at least 75% of the time.

(c) Use the probability density function to calculate the exact probability that the weekly repair cost will fall in the interval you calculated in part (b).

(d) Explain why it is acceptable to obtain a value different from 0.75 for part (c)

5. Suppose the service times (time from when you get to the window where you get your food to when you leave) for the drive-thru at McDonald's are normally distributed with a mean of 60 seconds and a standard deviation 15 seconds.

   (a) What is the probability it takes over 70 seconds to serve a randomly selected customer at the drive-thru?

   (b) What must the service time be under to be among the lowest 1% of service times?

   (c) The employees at McDonald's get a free lunch if the average service time for 100 consecutive cars is under 57 seconds. What is the probability the employees get a free lunch for the next 100 cars they serve?

6. An estimate of the mean recovery time for a new surgery is desired. 5 patients who underwent the surgery had the following recovery times (in days): 10, 7, 11, 12, 14.

   (a) Assuming these 5 patients represent a random sample, construct a 90% confidence interval for the population mean recovery time.

   (b) Are any assumptions about the population necessary for this confidence interval to be valid?

7. An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X = points earned on first part, Y = points earned on second part. Suppose the joint probability distribution of X and Y is given in the table below.

	Y			
	0	5	10	15
0	0.02	0.06	0.02	0.10
X 5	0.04	0.15	0.20	0.10
10	0.01	0.15	0.14	0.01

   (a) What percent of students got 10 points on the first part and 15 points on the second part?

   (b) Give the marginal probability distribution of Y.

   (c) Give the conditional distribution of X, given that Y = 5.

   (d) Are X and Y independent? Justify your response.

   (e) Without doing any calculations, do you think the correlation coefficient of X and Y will be positive or negative? Consider the definitions of these variables and the type of relationship that is likely to exist between them.

8. Answer the following questions.

(a) A personnel director for a corporation has hired ten new engineers. If three distinctly different positions are open at a particular plant, in how many ways can he fill the positions?

(b) Five cards are drawn without replacement from a standard 52 card deck. What is the probability of getting

i. 4 aces?

ii. 4 of a kind?

(c) 5 computer chips are drawn without replacement from a bin containing 10 good and 6 defective chips. What is the probability of getting exactly 2 defectives in the sample of size 5?

(d) Police records show that there are an average of 3 accidents per week on Route 40. Assuming the number of accidents in a week follows a Poisson distribution,

i. determine the probability of at least three accidents in a week

ii. give the formula for the probability mass function for the number of accidents in a year

9. A sample of 125 pieces of yarn had a mean breaking strength 6.1 Newtons (N) and standard deviation 0.7 N. A new batch of yarn was made, using new raw materials from a different vendor. In a sample of 75 pieces of yarn from the new batch, the mean breaking strength was 5.8 N and the standard deviation 1.0 N. Find a 90% confidence interval for the difference in mean breaking strength between the two types of yarn.

10. Explain the steps you would take in using a computer simulation to estimate the following probability. If $X \sim N(0, 1)$, find $P(X^2 > 1.5)$.

11. A random variable X has the probability density function $f(x) = 2e^{-2x}$ if $x > 0$.

(a) Find $P(X > 1)$

(b) Find $P(X > 2|X > 1)$

(c) Find the cumulative distribution function, $F(x)$

(d) Find the median of X. (Hint: Let m = the median, then $F(m) = 0.5$)

12. Answer the following true/false, multiple choice, and fill in the blank questions.

(a) If $X \sim N(1, 4)$ and $Y \sim N(10, 5)$, and X and Y are independent, what is the distribution of $X - Y$?

(b) A hypothesis test yields a p-value of 0.07. Circle all levels of significance for which H_0 will be rejected:

0.20	0.10	0.08	0.06	0.05	0.01

(c) In order to test $H_0 : \mu = 100$ versus $H_1 : \mu < 100$, a very large sample yields a mean of 101. At a 0.05 level of significance, would the null hypothesis be rejected? Why or why not?

(d) True or False. The confidence interval formula $\bar{X} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ can be used to estimate the population mean annual income based on data from a random sample of size 10.

(e) If the process of constructing a 95% confidence interval to estimate the mean of a population is repeated 1000 times. Approximately how many of these intervals would contain the mean?

(f) If $Var(X+Y) = 3$, $Var(X) = 1$ and $Var(Y) = 1.5$, find $Cov(X, Y)$.

(g) Find $z_{\frac{\alpha}{2}}$ for an 86% confidence level.

(h) If all other values remain the same, increasing the confidence level of a confidence interval will cause the margin of error to: (circle one)

decrease	remain the same	increase