Stat 50 – Worksheet #7: The Cumulative Distribution Function

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1. The continuous RV X has the probability density function (pdf) below.

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $P(X \leq 0.1)$

   (b) Find $P(X \leq b)$ where $0 < b < 1$

   (c) Find $P(X \leq 2.7)$

   (d) Find $P(X \leq b)$ where $b > 1$

   (e) Find $P(X \leq -3.3)$

   (f) Find $P(X \leq b)$ where $b < 0$

   (g) Define a new function $F(b) = P(X \leq b)$. Use parts (b),(d) and (f) to write a piecewise formula for $F(b)$ for all b in the real numbers. Note that $F(b)$ represents the cumulative area from $x = b$ on the right to $x \to -\infty$ on the left.

2. The random variable $X =$ the lifetime in years of a car battery The pdf of X is:

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0, \text{elsewhere} & \end{cases}$$

   (a) Graph $f(x)$

   (b) Find the cumulative distribution functiuon, $F(b)$, and graph it.

   (c) What is $F(x)$?

   (d) Verify that $F'(x) = f(x)$

3. (a) Find $\lim_{x\to\infty} F(x)$ in problem #1.

   (b) Find $\lim_{x\to\infty} F(x)$ in problem #2.

   (c) Is it true that for any cumulative distribution function,F, $\lim_{x\to\infty} F(x) = 1$. Justify your answer. (Hint: If f is a pdf, what is $\int_{-\infty}^{\infty} f(x)dx$ always equal to?)