Stat 50 – Worksheet #8: Joint Discrete Random Variables

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1. Let X represent the number of cars and Y the number of trucks that pass through a certain toll booth in a one-minute time interval. The joint probability mass function of X and Y is:

			Y (trucks)	
		0	1	2
	0	0.10	0.05	0.05
X(cars)	1	0.10	0.10	0.05
	2	0.13	0.25	?

   (a) What is the probably of 2 trucks and 2 cars in a one-minute interval?

   (b) What is the probability of no trucks in the one-minute interval?

   (c) Find the marginal distribution of Y, $p_Y(y)$.

   (d) Find the marginal distribution of X, $p_X(x)$

   (e) Find μ_X, μ_Y and σ_y^2.

   (f) Find the conditional distribution of X given $Y = 0$.

   (g) Find the conditional expectation $E(X|Y = 0)$ and interpret this value using plain English.

   (h) Find $P(X + Y = 3)$.

   (i) Find μ_{X+Y}. Interpret this mean using plain English.

   (j) Find μ_{XY}

   (k) Use your answers to parts (e) and (j) to see if $\mu_{XY} = \mu_X \cdot \mu_Y$

2. Suppose that X and Y are independent random variables with probability mass functions below.

X	0	1	2
$p_X(x)$	0.3	0.2	0.5

Y	0	1	2	3
$p_Y(y)$	0.1	0.2	0.4	0.3

   (a) Find μ_X and μ_Y

   (b) Find the joint distribution of X and Y. (Remember X and Y are independent so, for example, $p(0, 1) = P(X = 0, Y = 1) = p_X(0) \cdot p_Y(1) = 0.3(0.2) = 0.06$)

	Y			
	0	1	2	3
X=0		0.06		
X=1				
X=2				

   (c) Find the probability distribution of $W = XY$.

   (d) Find μ_W

   (e) Does $\mu_W = \mu_{XY}$ equal $\mu_X \cdot \mu_Y$?