Math 30 – Workshop #17

1. The graph of $f(x) = \frac{1}{\sqrt{x}}$ is shown below.

   (a) Find the equation for the tangent line to the graph of the function $f(x) = \frac{1}{\sqrt{x}}$ at $x = 4$.

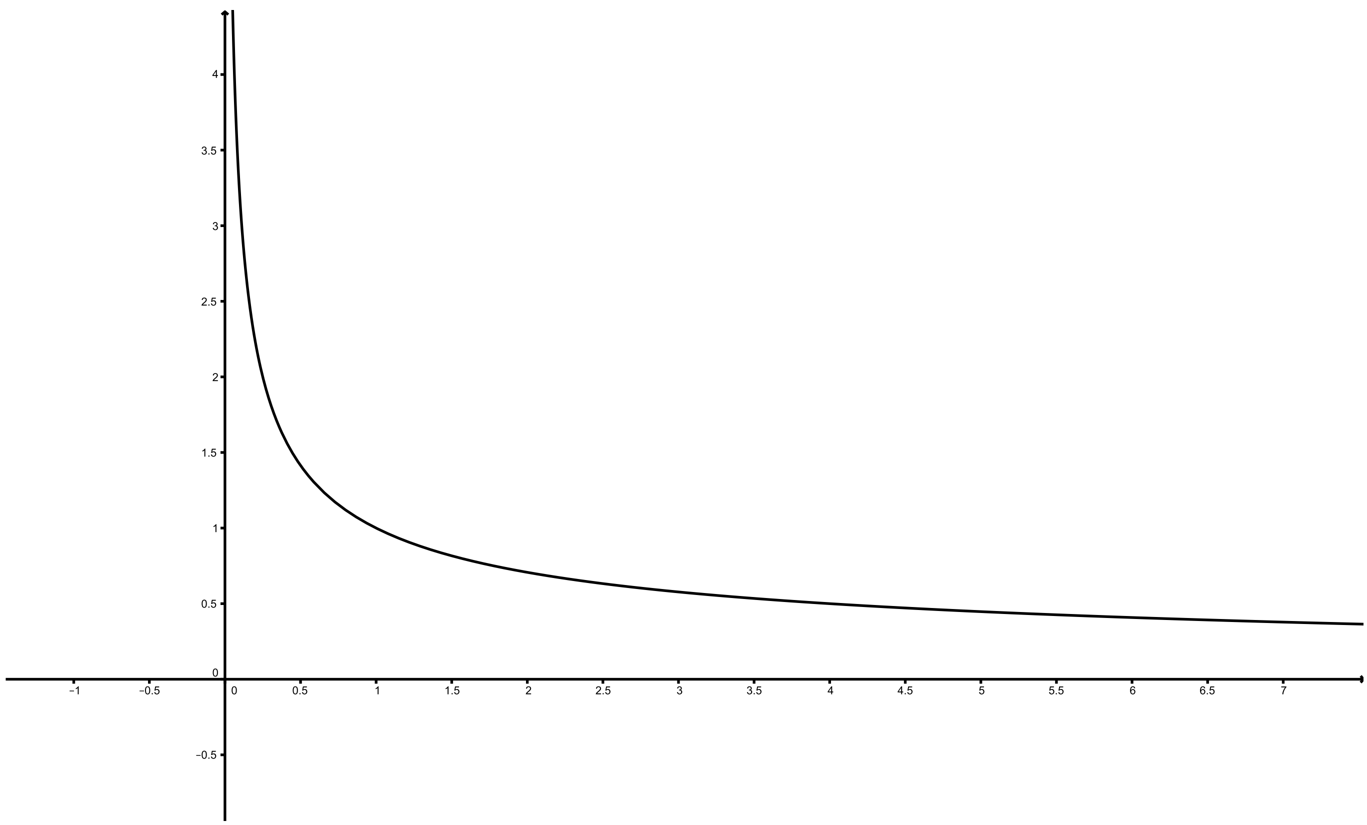
   (b) On the graph below, draw

   i. the tangent line from part 1a

   ii. a vertical distance that represents $\frac{1}{\sqrt{4.1}}$

   iii. a vertical distance that represents the tangent line value for y when x is 4.1.

   (c) Find the y-value of your tangent line from part 1a when $x = 4.1$. Now compare that with the value your calculator gives you for $\frac{1}{\sqrt{4.1}}$.



2. In this problem, we use a tangent line to a function to estimate the value of $28^{\frac{2}{3}}$.

   (a) Which of the following functions would be the most useful in obtaining a tangent line approximation to $28^{\frac{2}{3}}$? $f(x) = \sqrt{x}, f(x) = \dfrac{1}{x}$, or $f(x) = x^{\frac{2}{3}}$?

   (b) Find $f'(x)$ for your answer to 2a.

   (c) Find the equation of the tangent line at a value of x close to $x = 28$, but with a value of x that makes the calculation of the slope of the tangent line easy without a calculator.

   (d) Use the tangent line from 2c to estimate $28^{\frac{2}{3}}$.

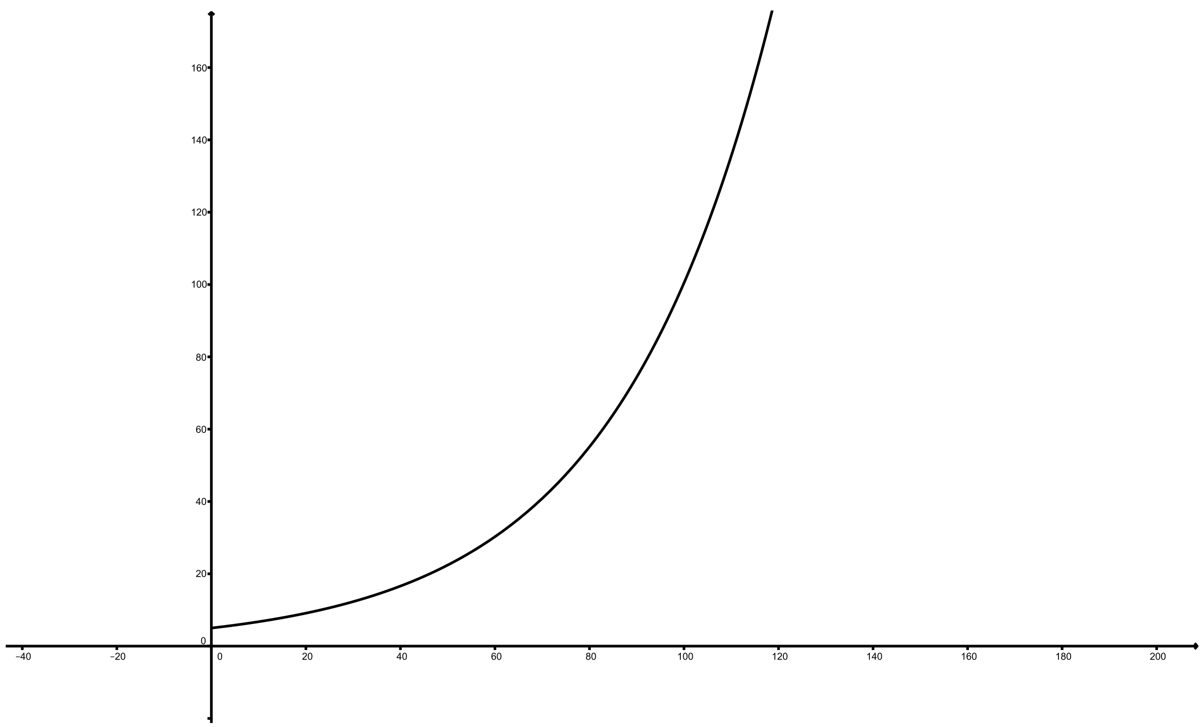
   (e) Calculate the exact value of $28^{\frac{2}{3}}$ using your calculator and compare it to the approximation in part 2d.

3. The graph below shows the population of bacteria in the petri dish x minutes after being added to a growth medium.

   (a) What do $f(80)$ and $f(100)$ equal? Write a sentence that explains what these numbers represent in the context of bacteria.

   (b) If someone did the computation $\dfrac{100.4 - 55.1}{100 - 80} = 2.27$ from this graph, what do you think they were computing?

   (c) If $f'(100) = 3.01$, write a sentence that explains what this number represents in the context of bacteria.



4. A rectangle is being drawn on a computer, by dragging one corner of the rectangle to change the dimensions of the rectangle. At the moment when it is 40 mm long and 30 mm wide, the length is increasing at a rate of 3 mm per second and the width is decreasing at the rate of 2 mm per second. At that moment, at what rate is the area of the rectangle changing (and is it increasing or decreasing in area)?