Math 30 – Workshop #20

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1. Sketch a graph that satisfies all of the following criteria.
   - $f(x)$ is continuous
   - $f(-4) = 0$ and $f(x)$ does not equal zero anywhere else
   - $f'(x) > 0$ on the intervals $(-\infty, -1)$ and $(2, \infty)$
   - $f'(x) < 0$ on the interval $(-1, 2)$
   - $f'(-1)$ is undefined and $f'(2)$ is undefined

2. Sketch a graph that satisfies all of the following criteria.
   - $f(x)$ is continuous
   - $f(-2) = 0$, $f(1) = 0$ and $f(x)$ does not equal zero anywhere else
   - $\lim_{x\to\infty} f(x) = 3$
   - $\lim_{x\to-\infty} f(x) = \infty$
   - $f'(x) < 0$ on the intervals $(-\infty, -1)$ and $(0, \frac{1}{2})$
   - $f'(x) > 0$ on the intervals $(-1, 0)$ and $(\frac{1}{2}, \infty)$

3. Find the absolute maximum and minimum of the following functions on the given interval.
   (a) $f(x) = \sin x$, $0 \leq x \leq \frac{\pi}{2}$
   (b) $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$
   (c) $f(x) = x - \ln x$, $[\frac{1}{2}, 2]$

4. Show that the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum or a local minimum.

5. For each of the problems below, sketch a graph of a function that is continuous on $[1, 5]$ and has the given properties.
   (a) Absolute minimum at 2, absolute maximum at 3, local minimum at 4
   (b) No local maximum or minimum, but 2 and 4 are critical numbers
   (c) Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4