Math 30 – Workshop #21

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1. For each of the following functions, determine the intervals of increase and decrease, find all local maxima and minima, determine intervals of concavity, and find all points of inflection. After completing this, graph the function on your calculator and check that your results agree with the graph you are seeing on your calculator.

   (a) $f(x) = 2x^3 - 9x^2 + 12x - 2$

   (b) $f(x) = (x^2 - 1)^2$

2. Sketch the graph of a function f for which all of the following are true:

   - $f''(x) > 0$ for $x > 1$ and $f''(x) < 0$ for $x < 1$
   - $f'(x) > 0$ for $x < -1$ and $x > 3$; $f'(x) < 0$ for $-1 < x < 3$

3. Consider $h(x) = \ln(1 + x^2)$. (Please do NOT graph this function on your calculator.)

   (a) Find all critical numbers (values of x where $h'(x)$ is zero or undefined).

   (b) Find all inflection points.

   (c) Use the second derivative test to determine the local maxima and minima.

   (d) Without using your calculator sketch a graph of the function showing all local extrema and inflection points.

4. Suppose $f(x)$ is a continuous function that has 4 zeros.

   (a) Is it possible that $f'(x)$ has less than 4 zeros? If so, sketch a possible graph of $f(x)$. If not, explain why not.

   (b) Is it possible that $f'(x)$ has more than 4 zeros? If so, sketch a possible graph of $f(x)$. If not, explain why not.

   (c) Is it possible that $f'(x)$ has exactly 4 zeros? If so, sketch a possible graph of $f(x)$. If not, explain why not.