Math 30 – Workshop #25

To do the first two exercises, you need to know the following:

$\displaystyle\int_a^b f(x)dx$ represents the area between the graph of $y = f(x)$ and the x-axis, between $x = a$ and $x = b$.

However, areas that are below the x-axis are counted as being negative.

1. Evaluate the following integrals by finding the corresponding areas.

   (a) $\int_0^3 2x dx$

   (b) $\int_1^3 (x+2)dx$

   (c) $\int_0^3 (x-1)dx$

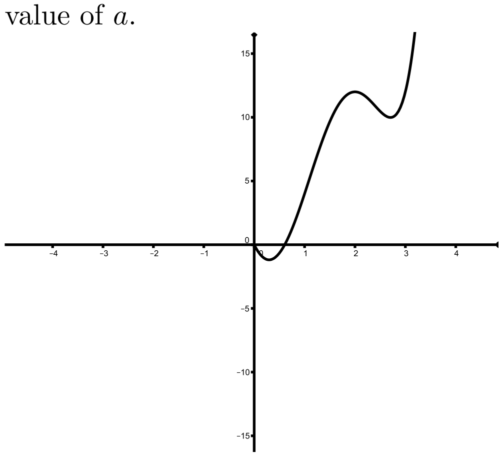
   (d) $\int_0^4 (3-2x)dx$

   (e) $\int_{-1}^1 \sqrt{1-x^2}dx$

2. (a) Compute $\int_{-a}^a x^3 dx$

   (b) Compute $\int_{-a}^a \sin(x)dx$

   (c) Below is a partial sketch of the graph of $f(x)$. Complete the graph, so that $\int_{-a}^a f(x)\ dx = 0$ for any value of a.



   (d) Find conditions on a function $f(x)$ under which $\int_{-a}^a f(x)dx = 0$ for all a.

   (e) Compute $\int_{-a}^a xe^{x^2} dx$

   (f) What property of a function $g(x)$ would ensure that $\int_{-t}^t g(x)dx = 2\int_0^t g(x)dx$ for every value of t?

3. Let $f(x) = \frac{x+1}{x^2}$

   (a) Find the intervals on which f is increasing.

   (b) Identify any local maxima or minima.

   (c) Find the regions in which the graph of f is concave downward.

   (d) Find all inflection points for the graph of f.