

1. Compute the following limits:

(a) $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$

(b) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$

(c) $\lim_{x \rightarrow 0^+} x e^{\frac{1}{x}}$

(d) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{1 - \cos x}\right)$

2. (a) Find a function $f(x)$ such that $f'(x) = x^2 - 5$.

(b) Find a function $f(x)$ such that $f'(x) = x^2 - 5$ and $f(3) = 2$.

(c) How many functions are there such that $f'(x) = x^2 - 5$?

3. If $f''(x) = \frac{1}{x^2} + 4x^2$ and $f'(1) = 1$ and $f(1) = 5$, then find $f(x)$.

4. The girth of a shipping carton is the perimeter of an end. Shipping restrictions require that the sum of the girth and length not exceed 100 in. Find a rule that describes the volume of a package with a square end (the sum of whose girth and length equals 100 in) as a function of the width of an end. Then use calculus to find the maximum volume, and the dimensions that give that volume.

5. Differentiate the following.

(a) $f(x) = \frac{x^2 + 3}{\tan x}$

(b) $g(x) = \cos(\ln(\cos x))$

(c) $h(x) = e^{(xe^{2x})}$