Math 31 – Workshop #11

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1. A right triangle has a side of length $\sqrt{4-x^2}$ and a side of length x, neither of which are the hypotenuse. What is the length of the other side? Draw a labeled right triangle.

2. A right triangle has a side of length $\sqrt{4+x^2}$ and a side of length 2, one of which is the hypotenuse. What is the length of the other side? Draw a labeled right triangle.

3. A right triangle has a side of length $\sqrt{x^2-4}$ and a side of length 2, neither of which is the hypotenuse. What is the length of the other side? Draw a labeled right triangle.

4. Consider the integral below.

$$\int \frac{1}{\sqrt{4-x^2}}\,dx$$

   (a) Look at your triangle from Problem 1. There are two non-right angles, for which of the angles will $\sin u = \frac{x}{2}$? Label that angle u.

   (b) If $\sin u = \frac{x}{2}$, then $x = 2\sin u$. What does dx equal?

   (c) Rewrite the above integral in terms of u.

   (d) Use the above substitution to compute $\int \frac{1}{\sqrt{4-x^2}}\,dx$.

5. In the above problem we used the fact that $\sin u = \frac{x}{2}$ to help us figure out $\int \frac{1}{\sqrt{4-x^2}}\,dx$.

   (a) If the integral was $\int \frac{1}{\sqrt{4+x^2}}\,dx$ instead, we would use the triangle in problem 2.

   In your triangle for Problem 2 label the angle u for which $\tan u = \frac{x}{2}$.

   (b) Make this substitution to get the integral in terms of u. Why is this substitution useful?

   (c) If the integral was $\int \frac{1}{\sqrt{x^2-4}}\,dx$ instead, we would use the triangle in problem 3.

   In your triangle for Problem 3 label the angle u for which $\sec u = \frac{x}{2}$.

   (d) Make this substitution to get the integral in terms of u. Why is this substitution useful?

Compute the following integrals.

6. $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

7. $\int \frac{\sqrt{x^2-1}}{x^3}\,dx$

8. $\int \frac{1}{e^x\sqrt{1-e^{2x}}}\,dx$