

Math 31 – Workshop #21

1. Consider the series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$.
 - (a) The fifth partial sum is $s_5 = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$. Write out the terms that would appear in the expression $\frac{1}{3}s_5$.
 - (b) Looking at what you have written above, what terms would be left in the expression $s_5 - \frac{1}{3}s_5$?
 - (c) Use the above expressions to determine what the numeric value of s_5 is. (Do not just add up the first five terms.)
 - (d) The n th partial sum is $s_n = 1 + \frac{1}{3} + \frac{1}{9} + \dots + \frac{1}{3^{n-2}} + \frac{1}{3^{n-1}}$. Write out the first few and last few terms that would appear in $\frac{1}{3} \cdot s_n$.
 - (e) Looking at what you have written above, what terms would be left in the expression $s_n - \frac{1}{3}s_n$?
 - (f) Use the above expressions to determine a formula for s_n .
 - (g) What is the limit as n goes to infinity of s_n ?

2. Consider the series $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \dots$.
 - (a) The fifth partial sum is $s_6 = 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625} + \frac{2}{3125}$. Write out the terms that would appear in the expression $\frac{1}{5}s_6$.
 - (b) Looking at what you have written above, what terms would be left in the expression $s_6 - \frac{1}{5}s_6$?
 - (c) Use the above expressions to determine what the numeric value of s_6 is. (Do not just add up the first six terms.)
 - (d) The n th partial sum is $s_n = 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \dots + \frac{2}{5^{n-2}} + \frac{2}{5^n}$. Write out the first few and last few terms that would appear in $\frac{1}{5} \cdot s_n$.
 - (e) Looking at what you have written above, what terms would be left in the expression $s_n - \frac{1}{5}s_n$?
 - (f) Use the above expressions to determine a formula for s_n .
 - (g) What is the limit as n goes to infinity of s_n ?

3.
 - (a) In problem #1, why did we multiply s_n by $\frac{1}{3}$?
 - (b) In problem #2, why did we multiply s_n by $\frac{1}{5}$?
 - (c) For which of the following series could we use a method similar to the above two problems in order to find the sum of the series?

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$$\frac{5}{3} + \frac{5}{6} + \frac{5}{12} + \frac{5}{24} + \dots$$

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \dots$$

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots$$

- (d) Find the sum of the series you indicated above.