Math 31 – Workshop #27

1. Recall that the formula for the Maclaurin series is as follows.

$$f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+\cdots$$

   (a) Use this formula to find the Maclaurin series for $f(x)=\cos(2x)$ through the degree 4 term.

   (b) On a graphing calculator, graph both $f(x)=\cos(2x)$ and the degree four Maclaurin polynomial that you found above. On what interval would you say the polynomial is a good approximation of $f(x)$?

   (c) Recall that the Maclaurin series for $\cos x$ is as follows.

$$\cos x=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\cdots$$

   Use this to find the Maclaurin series for $f(x)=\cos(2x)$. Does this match what you found in part 1a?

2. Find the Maclaurin series for the function $f(x)=(x+2)^2-3$.

3. The power series representation of $f(x)=\sqrt{1-x}$ is given below, and it converges for $-1<x<1$.

$$\sqrt{1-x}=1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3-\frac{5}{128}x^4-\cdots-\frac{1\cdot 3\cdot 5\cdots(2n-3)}{n!2^n}x^n-\cdots$$

   (a) Using this series, find the first 4 terms in the power series for $-\frac{1}{2}(1-x)^{-\frac{1}{2}}$.

   (b) Use this to find the first four nonzero terms in the power series for $\frac{1}{\sqrt{1-x^2}}$.

   (c) Use this to find the first four nonzero terms in the power series for $\arcsin x$.

   (d) Use this to find the first four nonzero terms in the power series for $\frac{\arcsin x}{x}$.

   (e) Using what you have found thus far, find $\lim\limits_{x\to 0}\frac{\arcsin x}{x}$.

   (f) Check your answer to the above limit by using L'Hôpital's Rule.